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Vector Function Inverse Riemann Boundary Value Problem and Its Solving

Ding Yun[1] and Yang Xiaochun[1*]

[1] *Department of Mathematics, Dalian Maritime University, Dalian, 116026, P. R. China.*

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Abstract

In the paper, generally,we solve a irregular vector functions inverse Riemann boundary value problem(R-problem). The kernel of our method is to regularize those equations via introducing some diagonal matrices. Then, we get the solution of the problem at the end of the paper.

Keywords: *Vector function; irregular; inverse problem; riemann boundary value problems; general solution.*

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1 Introduction

A lot of results about regular or irregular Riemann boundary value problem were systematically introduced and collected in references[1-4]. We need to point out that, in recently twenty years, Lu Jianke, a famous scholar and mathematician in China, and his group has done a lot of research in the region, especially on periodic Riemann boundary value problem and double periodic Riemann boundary value problem. Their results are collected in Lu's book[1]. Moreover, in those works, Lin Yuba has done a series of research which are very systematic on irregular Riemann boundary value problem, (whose works was published on the Transaction of Wuhan University or Yunnan Normal University). Refs [5-10] ranks only the works directly related to what we should have done in this paper.

Corresponding author: E-mail: yangxiaochun@dlmu.edu.cn

Compared to the study of Riemann boundary value problem, it seems that the study of a vector function Riemann boundary value problem developed slowly in past several decades, particularly on irregular Riemann boundary value problem.

Refs [11] gives some constructive methods, but the solving process was still on the stage of description. And there is no tangible analytic expression stated for cleaning up the isolated zeros and singularities in the book. Meanwhile, the process of the cleaning up isolated zeros and singularities may change the partial index of the matrix, which will greatly affect the solving of the problem and the solution expression of the problem will change as well.

Matrix theory is the important tool for solving the inverse of Riemann boundary value problem. Many matrix calculi rely on determinant calculation, whose operations are quite different from that of matrix. Because many isolated zeroes and singularities on the matrix are required to be cleaned up in solving process, and finding and cleaning up those on the matrix have to use determinant operations, the research work is very difficult.

To overcome the difficulty of the partial index problem in solving [1-3], paper [8] put forward an effective method, through which the original problem is transformed to an equivalent problem by introducing some diagonal matrices. Then the solving only needs to determine the total index of the matrix but the partial index. Furthermore, papers [9-10] also give a general method, which greatly help to solve the irregular vector function Riemann or Hilbert boundary value problem, to cope with the situation that the determinant of a matrix equals to zero.

The study in refs [12] shows a problem seems related to the inverse of Riemann boundary value problem, which was proposed by Ioakimidis. Li had done some further research [13]. Nevertheless Riemann boundary value inverse problem was formally named by Lu et al. and was successively supported by national natural of science funds of China (three times). Of course, the new mention in Lu et al. overcome some defects in logic that implied in the past works[12-13].

Obviously, a vector function Riemann boundary value inverse problem is a new topic. Until now, there is no result that can be seen about it.

In this paper, a vector function Riemann boundary value inverse problem is discussed. In the solving, we are fully making use of the specialty and difference between matrix and determinant. By introducing a diagonal matrix, we transfer the irregular vector function Riemann boundary value inverse problem to a regular one. Then, by using the result in paper [5], we obtain the general solution of the question.

2 Illumination

The paper studies the regular or irregular vector Riemann boundary value inverse problem.

For simple reason, we suppose the L is a smooth closed contour in the complex plane oriented counter-clockwise. Notes D^+ as the curve's interior, D^- as the curve's exterior, and $0 \in D^+$. Denotes

$$\varphi(z) = \begin{pmatrix} \varphi_1(z) \\ \varphi_2(z) \\ \vdots \\ \varphi_n(z) \end{pmatrix} = \begin{pmatrix} \varphi_1(z) & \varphi_2(z) & \cdots & \varphi_n(z) \end{pmatrix}^T \tag{1.1}$$

Problem Description Irregular vector function inverse R-problem is finding a pair of vector functions $(\Phi(z), H(t))$ as form (1.1) with jumps on L, satisfying

$$\Phi^+(t) = G_1(t)\Phi^-(t) + g_1(t)H(t), \quad t \in L \tag{1.2}$$

$$\Phi^+(t) = G_2(t)\Phi^-(t) + g_2(t)H(t), \quad t \in L \tag{1.3}$$

where $G_p(t) = (G^p_{jk}(t))_{n\times n}(p = 1, 2)$ is a known $n \times n$ function matrix on L,$\in H$, $g_p(t) = (g^p_{jk}(t))_{n\times n}(p = 1, 2)$ is a another known $n \times n$ function matrix ($\in H$) as form. $\Phi(z) = (\Phi_1(z), \Phi_2(z), \cdots, \Phi_n(z))^T$ is an unknown sectional holomorphic vector function, which is belonging to H on L also. Suppose the singular order of $\Phi(z)$ at ∞ is finite.$H(t)$ is another unknown sectional holomorphic vector function, $\in H$ on L. Without losing of generality, we just solve the problem in class R_0 here.

Compared with the regular Riemann boundary value problem, $\det G(t)(t = 1, 2)$ can have several zero points on L, else $\det[g_2(t) - g_1(t)]$ and $\det[g_2(t)G_1(t) - g_1(t)G_2(t)]$. For simplicity, it still supposes $\det G(t) \neq 0(t = 1, 2)$.

Supposed the zero points of $\det[g_2(t) - g_1(t)]$ on L are $\sigma_1, \sigma_2, \cdots, \sigma_l$, and the order of their is $\sigma_1, \sigma_2, \cdots, \sigma_l$, respectively. Notes

$$K(t) = \prod_{s=1}^{l} (t - \sigma_s)^{\varepsilon_s},$$

and let $T(t) = (T(t))_{n\times n}$ a diagomal matrix as follows

$$T(t) = diag(1, 1, \cdots, 1, 1/K(t)),$$

and $T^*(t) = (T^*(t))_{n\times n}$ another diagomal matrix,

$$T(t) = diag(1, 1, \cdots, 1, K(t)).$$

3 Problem Solving

Left-multiply two side of (1.2) by $g_2(t)$, and left-multiply two side of (1.3) by $g_1(t)$. The sum of these two equation is

$$\{g_2(t) - g_1(t)\}\Phi^+(t) = \{g_2(t)G_1(t) - g_1(t)G_2(t)\}\Phi^-(t), \quad t \in L. \tag{2.1}$$

For solving problem (2.1), first of all, we left-multiply the diagonal matrix $T(t)$ on it. It is easy to prove that $\det\{T(t)[g_2(t) - g_1(t)]\} \neq 0$ now. Continuously, left multiplied matrix $\det\{T(t)[g_2(t) - g_1(t)]\}^{-1}$ on the two sides of the equation, we have

$$\Phi^+(t) = \tilde{G}(t)\Phi^-(t), \quad t \in L. \tag{2.2}$$

here,

$$\tilde{G}(t) = \{T(t)[g_2(t) - g_1(t)]\}^{-1}T(t)\{g_2(t)G_1(t) - g_1(t)G_2(t)\}, \quad t \in L. \tag{2.3}$$

Suppose the zero points of $\det \tilde{G}(t)$ and $1/\det \tilde{G}(t)$ on whole plane are $\alpha_1, \alpha_2, \cdots, \alpha_p$ and $\beta_1, \beta_2, \cdots, \beta_l$ $(\alpha_j \neq \beta_q, j = 1, 2, \cdots, p; k = 1, 2, \cdots, q)$, respectively. Their order is $\lambda_1, \lambda_2, \cdots, \lambda_p$, and $\mu_1, \mu_2, \cdots, \mu_q$. Noted

$$\Pi_1(z) = \prod_{j=1}^{p} (z - \alpha_j)^{\lambda_j}, \quad \Pi_2(t) = \prod_{j=1}^{q} (z - \alpha_j)^{\mu_j}, \quad \Pi(t) = \Pi_1(t)/\Pi_2(t)$$

$$\lambda = \sum_{j=1}^{p} \lambda_j, \qquad \mu = \sum_{j=1}^{q} \mu_k,$$

$$D_{n\times n}(t) = diag(1, 1, \cdots, 1, 1/\Pi(t)), \quad D^*_{n\times n}(t) = (1, 1, \cdots, 1, \Pi(t))$$

Lets

$$G_0(t) = D^*(t)\tilde{G}(t), \quad t \in L. \tag{2.4}$$

then

$$\det G_0(t) \neq 0, \quad 1/\det G_0(t) \neq 0.$$

Obviously, problem (2.2) is transferred to

$$D_1(t)\Phi^+(t) = G_0(t)D_2(t)\Phi^-(t), \quad t \in L \qquad (2.5).$$

where $D_1(t) = diag(1, 1, \cdots, 1, \Pi_2(t))_{n\times n}$ and $D_2(t) = diag(1, 1, \cdots, 1, \Pi_2(t))_{n\times n}$.

Introducing a new sectional holomorphic vector function:

$$\Psi(z) = \begin{cases} D_2(t)\Phi(z), & z \in D^+ \\ D_1(t)\Phi(t), & z \in D^- \end{cases} \tag{2.6}$$

then, problem (2.2) is transferred to

$$\Psi^+(t) = G_0(t)\Psi^-(t) \quad t \in L \tag{2.7}$$

It is a regular vector function Riemann boundary value problem whose canonical solution matrix could be given by refs [1], in which we note the canonical solution matrix as $X(z)$, then,

$$X^+(t) = G_0(t)X^-(t), \quad G_0(t) = X^+(t)[X^-(t)]^{-1}. \quad (t \in L) \tag{2.8}$$

Consequently

$$[X^+(t)]^{-1}\Psi^+(t) = [X^-(t)]^{-1}\Psi^-(t) \quad t \in L \tag{2.9}$$

Since the order of $\Phi(z)$ at ∞ is finite, by the Liouville theorem, we can obtained that the solution of problem (2.1) in R_0 is

$$\Phi(z) = \begin{cases} D_2^*(z)(P_{\kappa+\lambda}(z)X(z)) \\ D_1^*(z)(P_{\kappa+\lambda}(z)X(z)) \end{cases} = \begin{cases} D_2^*(z)\sum_{k=1}^{n} P_k(z)X^k(z), & z \in D^+ \\ D_1^*(z)\sum_{k=1}^{n} P_k(z)X^k(z), & z \in D^- \end{cases} \tag{2.10}$$

where, κ is a index of matrix $G_0(t)$(seeing refs. [1-3]); $D_1^*(t) = diag(1, 1, \cdots, 1, 1/\Pi_2(t))_{n\times n}$ and $D_2^*(t) = diag(1, 1, \cdots, 1, 1/\Pi_2(t))_{n\times n}$; $X(z) = (X^1(z), X^2(z), \cdots, X^n(z))$; $P_{\kappa+\lambda}(z) = (P_1(z), P_2(z), \cdots, P_n(z))$, all $P_k(z)(k = 1, 2, \cdots, n)$ are polynomial which the order is no more then $\kappa + \lambda$.

Because the vector function $\Phi^+(t)$ and $\Phi^-(t)$ should be bounded on L, the last terms in polynomial vector must be taken as $P_{\kappa+\lambda}(z)$

$$P_n(z) = D_1(z)D_2(z)P_{\kappa-\mu}(z). \tag{2.11}$$

That is

$$\Phi(z) = \begin{cases} D_2^*(z)(P_{\kappa+\lambda}(z)X(z)) \\ D_1^*(z)(P_{\kappa+\lambda}(z)X(z)). \end{cases} \tag{2.12}$$

The research method could be extended to the class R_m , especially in R_{-1} and the result be obtained very easy. In fact, now, the result almost the same to the theorem 1. The only difference is that the order of polynomial vector here is not equal to $\kappa + \lambda$.

Afterward, taking $\Phi(z)$ into (1.2) or (1.3), then, we can get $H(t)$:

$$H(t) = g_1^{-1}(t)\{D_2(z)P_{\kappa-\mu}(t)X(t) - G_1(t)D_1(z)P_{\kappa-\mu}(t)X(t)\}. \tag{2.13}$$

$$= g_2^{-1}(t)\{D_2(z)P_{\kappa-\mu}(t)X(t) - G_2(t)D_1(z)P_{\kappa-\mu}(t)X(t)\}. \tag{2.14}$$

Thereby, a closed form solution of the problem is obtained.

Theorem 1 Solving homogeneous problem (2.1) in R_0 , if $\kappa \geq \mu$, then, the generalized solution is $(\Phi(z), H(t))$, in which the $(\Phi(z), H(t))$ is given by (2.12) and (2.13) or (2.14); $P_{\kappa+\lambda}(z)$ is a polynomial vector that the order of every component $P_k(z)$ (k=1, , n-1) is no more than $\kappa + \lambda$ and $P_n(z) = D_1(z)D_2(z)P_{\kappa-\mu}(z)$; and if $\kappa \leq \mu$, then, all results is kept except the $P(z)$ should be taken to 0.

Though the theorem 1 provides the solution of the problem, the expressions of (2.12), (2.13) or (2.14) can't be used directly because of the canonical solution matrix related to the partial indexes. In other words, if one wants getting the canonical solution matrix, first of all, he must finish two things: one is to remove all zero points of the basic solution matrix and get the normal solution matrix; the other is to determine the partial indexes. As we all known, there is no method or formula to calculate partial indexes. Hence, the (2.12), (2.13) or (2.14) is only a closed form solution rather a analytic solution.

For solving the problem, here, we directly solve the problem from the express (2.7)and give the analytic solution at last. The solving process are as follows.

By the definition of index in refs. [1], it has

$$\kappa = \frac{1}{2\pi i}[\log\det G_0(t)]_L = \frac{1}{2\pi}[\arg\det G_0(t)]_L. \tag{2.15}$$

Lets $\Lambda(z) = diag(1, 1, \cdots, 1, z^{-\kappa})_{n\times n}$; $\Lambda(z) = diag(1, 1, \cdots, 1, z^{-\kappa})_{n\times n}$, and notes

$$G_e(t) = \Lambda(t)G_0(t), \tag{2.16}$$

and complement define that the function value of $G_e(t)$ at removable isolated singular point is taken its limiting value. Then, the problem (2.7) is transferred to

$$\Psi^+(t) = \Lambda^*(t)G_e(t)\Psi^-(t), \tag{2.17}$$

Introducing function

$$\Omega(z) = \begin{cases} \Psi(z), & z \in D^+ \\ \Lambda^*(z)\Psi(z), & z \in D^- \end{cases} \tag{2.18}$$

thus, the problem is varied as a equivalence problem

$$\Omega^+(t) = G_e(t)\Omega^-(t), \quad t \in L. \tag{2.19}$$

But, in this time, the index of problem (2.19) is

$$\kappa = \frac{1}{2\pi i}[\log\det G_e(t)]_L = \frac{1}{2\pi}[\arg\det G_e(t)]_L = 0. \tag{2.20}$$

Theorem 2 Solving problem of the regular functions R-problem (1.2) in R_λ can be transformed to an equivalent regular functions R-problem (2.19) in R_κ, in which, the index of the problem is equal to zero.

Solving the regular vector function R-problem (2.19), we can get the basic solution matrix (seeing refs [1]). We had proved in our other paper that, for the problem (2.19), the normal solution matrix was the canonical solution matrix (seeing refs [1,2]) and the normal solution matrix could be obtained by multiplied a special diagonal matrix. (seeing refs [8]). Here, we do not repeat it. Hence, the partial index is no use here.

Solving regular vector function R-problem (2.19), we can get an explicit canonical solution matrix:

$$X_e^+(t) = G_e(t)X_e^-(t), \quad or \quad G_e(t) = X_e^+(t)[X_e^-(t)]^{-1}. \tag{2.21}$$

The problem (2.19) can be written as

$$[X_e^+(t)]^{-1}\Omega^+(t) = [X_e^-(t)]^{-1}\Omega^-(t), \quad t \in L \tag{2.22}$$

Let $F(z) = [X_e(z)]^{-1}\Omega(z)$, then, it is a holomorphic vector on whole complex plane. Since the solving for the equivalent problem is process in class $R_{\kappa+\lambda}$, the Liouville theorem show that it is only a polynomial vector which is denoted by $P(z)$,

$$P(z) = (P_1(z), P_2(z), \cdots, P_n(z))^T, \tag{2.23}$$

the order of $P(z)$ is no more than $\kappa + \lambda$.

Taking it to the original problem, we can get the expression of the general solution of (2.7) in R_λ, which is

$$\Psi(z) = \begin{cases} P(z)X_e(z) \\ \Lambda(z)P(z)X_e(z) \end{cases} = \begin{cases} \sum_{k=1}^n P_k(z)X_e^k(z), & z \in D^+ \\ \Lambda(z)\sum_{k=1}^n P_k(z)X_e^k(z), & z \in D^- \end{cases} \tag{2.24}$$

where,$X(z) = (X^1(z), X^2(z), \cdots, X^n(z))$, and $P_k(z)(k = 1, , n-1)$ is polynomial whose order is no more then $\kappa + \lambda$.

Taking $\Psi(z)$ into the express (2.6), we have

$$\Phi(z) = \begin{cases} D_2^*(z)P(z)X_e(z) \\ D_1^*(z)\Lambda(z)P(z)X_e(z) \end{cases} = \begin{cases} D_2^*(z)\sum_{k=1}^n P_k(z)X_e^k(z), & z \in D^+ \\ D_1^*(z)\Lambda(z)\sum_{k=1}^n P_k(z)X_e^k(z), & z \in D^- \end{cases} \tag{2.25}$$

Similarly, for keeping the vector function $\Phi^+(z)$ and $\Phi^-(z)$ are bounded on complex plane, the last terms in polynomial vector $P(z)$ must be taken as

$$P_n(z) = D_1(z)D_2(t)(P_{\kappa-\mu}(z) \tag{2.26}$$

$H(t)$ is still given by express (2.13) or (2.14) but the $\Phi(z)$ in here will replace by (2.25).

Summary up the conclusion about, we have

Theorem 3 Solving Irregular vector R-problem (1.2),(1.3) solving in R_0 , if $\kappa \geq \mu$, then, the generalized solution is $(\Phi(z), H(t))$, in which the $(\Phi(z), H(t))$ is given by (2.25) and (2.13) or (2.14) but the $\Phi(z)$ in here will be replaced by (2.25); $P(z)$ is a polynomial vector that the order of every component $P_k(z)$ (k=1, , n-1) is no more than $\kappa + \lambda$ and $P_n(z) = D_1(z)D_2(z)P_{\kappa-\mu}(z)$; and if $\kappa \leq \mu$, then, all results is kept except the $P_n(z)$ should be taken to 0.

4 Numerical Example

Here, we give some examples for the using.

Example 1 Regular the following problem:

$$\Phi^+(t) = \begin{pmatrix} \frac{t-1}{t(2t+1)} & 1 \\ 0 & 2t-1 \end{pmatrix} \Phi^-(t) + \begin{pmatrix} t+i & \frac{1}{2t+1} \\ 2t+1 & 0 \end{pmatrix} H(t) \tag{3.1}$$

and

$$\Phi^+(t) = \begin{pmatrix} 0 & \frac{2t}{2t+1} \\ \frac{(t-1)(4t-3)}{2t-1} & 3t-2it+1 \end{pmatrix} \Phi^-(t) + \begin{pmatrix} 2t & \frac{1}{2t-1} \\ 2t+1 & 1 \end{pmatrix} H(t). \tag{3.2}$$

Solution Now

$$G_1(t) = \begin{pmatrix} \frac{t-1}{t(2t+1)} & 1 \\ 0 & 2t-1 \end{pmatrix}, \quad G_2(t) = \begin{pmatrix} 0 & \frac{2t}{2t+1} \\ \frac{(t-1)(4t-3)}{2t-1} & 3t-2it+1 \end{pmatrix}$$

and

$$g_1(t) = \begin{pmatrix} t+i & \frac{1}{2t+1} \\ 2t+1 & 0 \end{pmatrix}, \quad g_2(t) = \begin{pmatrix} 2t & \frac{1}{2t-1} \\ 2t+1 & 1 \end{pmatrix}.$$

Since $\det(g_2(t) - g_1(t)) = t - i$, the $T(t)$ is taken as $diag(1, (t-i)^{-1})$. Further,

$$[T(t)g_2(t) - g_1(t)]^{-1} = \begin{pmatrix} \frac{1}{t-i} & \frac{-2}{4t^2-1} \\ 0 & t-i \end{pmatrix},$$

and

$$\tilde{G}(t) = \{T(t)[g_2(t) - g_1(t)]\}^{-1}T(t)\{g_2(t)G_1(t) - g_1(t)G_2(t)\} = \begin{pmatrix} \frac{1}{t-i} & \frac{-2}{4t^2-1} \\ 0 & t-i \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{t-i} \end{pmatrix} \begin{pmatrix} \frac{t-1}{4t^2-1} & t \\ \frac{t-1}{t} & 2t \end{pmatrix}.$$

$\det \tilde{G}(t)$ has one zero point $t = 1$ and a isolate singular point $t = i$, the order of both is one. Therefore, we have $D^*(t) = diag1, (t-1)/(t-i)$ and

$$G_0(t) = D^*(t)\tilde{G}(t) = \begin{pmatrix} \frac{1}{t-i} & \frac{-2}{4t^2-1} \\ 0 & t-i \end{pmatrix} \begin{pmatrix} \frac{t-1}{4t^2-1} & t \\ \frac{1}{t} & \frac{t}{t-1} \end{pmatrix}, \quad t \in L. \tag{3.3}$$

Obviously, $\det G_0(t) \neq 0, \quad 1/\det G_0(t) \neq 0$, the problem is regularized now.

Continually, we solve the regular vector function Riemann boundary value problem

$$\Psi^+(t) = G_0(t)\Psi^-(t) \quad t \in L$$

and write out the solution. Since $G_0(t)$ is a rational function matrix, its basic solution can be solve directly. It is easy to find both $z_1 = 1/2$ and $z_2 = -1/2$ is isolated singular point with one order in $|z| < 1$, $z_3 = -\frac{1}{8} + i\frac{\sqrt{15}}{8}$ and $z_4 = -\frac{1}{8} - i\frac{\sqrt{15}}{8}$ is two zero point with one order in $|z| < 1$. The total index is zero. Furthermore,the normal solution matrix is

$$\Psi(z) = \{ \begin{matrix} \Psi^+(z) = G_0(z) \\ \Psi^-(z) = E \end{matrix} \tag{3.4}$$

and the canonical solution matrix is

$$\Psi(z) = \{ \begin{matrix} \Psi^+(z) = diag(1, 1/(z-z_3)(z-z_4))G_0(z) \\ \Psi^-(z) = diag(1, 1/(z-z_3)(z-z_4))E \end{matrix} \tag{3.5}$$

where, E is an unit matrix. Backs to $\Phi(z)$, it is

$$\Phi(z) = \{ \begin{matrix} \Phi^+(z) = diag(1, \frac{z-1}{(z-i)(z-z_3)(z-z_4)})G_0(z) \\ \Phi^-(z) = diag(1, \frac{z-1}{(z-i)(z-z_3)(z-z_4)})E \end{matrix} \tag{3.6}$$

and complement defined the function value at moving singularity is taken the limit value of the function, the solution is obtained.

5 Conclusion and Discussion

The kennel of this paper is that we introduce some special diagonal matrix to change the original vector function of inverse R-problem to a kind of regular vector functions of R-problem. Meanwhile, the method is helpful for removing the isolate zero points or isolated singular point.

In our paper, some special diagonal matrices are introduced. Though the matrices are defined as a fixed form, the constructed form isn't uniqueness. Actually, the researcher could arrange the element of diagonal matrix according to their requirement. The only need obey rule is keeping the determinant value fixed. What is easy for using, maybe rely on owns habit.

Though the solving is just for $G_0(t)$ but $G(t)$, it would be not increased the difficulty. In fact, the solving procession and difficulty for both matrices are almost the same of theoretically, the solving of the problem depend on the partial index[1,2] in the past. But there's no effective way to calculate the partial index by far. Through our way in this paper and the proof in refs [8], we don't have to rely on partial index. So the method of solving in the paper is more practical.

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Competing Interests

The authors declare that no competing interests exist.

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