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A Simulation Analysis of Distortion Operators of Wang, NIG and Cauchy: A Comparative Approach

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Abstract

The problem of pricing contingent claims has been extensively studied for non-Gaussian models and in particular, Black- Scholes formula has been derived for the NIG asset pricing model. This approach was originally studied in Insurance pricing where the distortion function was defined in terms of the normal distribution. It was also used to compare the standard Black-Scholes contingent pricing and distortion based contingent pricing. So, in this paper, we aim at using the Cauchy simulation analysis via MATLAB to compare the Wang distortion and NIG distortion operator with their pricing model. The results show that we can recuperate the Black-Scholes and NIG pricing model using the simulation of Cauchy distortion operator.

Keywords: Wang distortion operator; NIG; Cauchy distortion operator and simulation analysis.

1 Introduction

Financial mathematics has over the years made a great impact on the financial industry. The foundation of financial mathematics as it is known today has its origin in the seminal paper by [1], where the Ito's formula has been used for deriving a compact pricing formula for a standard European call option by formulating explicitly the model on the risk neutral measure under a set of assumptions.

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Option valuation is one of the most important topics in financial mathematics. The accurate modeling of financial price series is of paramount importance for the pricing of financial derivative such as option. To price derivatives securities, it is crucial to have a good modeling of the probability distribution of the underlying product. The most famous continuous time model used is the calibrated Black-Sholes model. It uses the Normal distribution to fit the log-returns of the underlying; the price process of the underlying is given by the geometric Brownian motion:

$$X_t = X_0 \, exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t\right)$$

where $(B_t, \ t \geq 0)$ is a standard Brownian motion, i.e B_t follows a Normal distribution with mean 0 and variance t. It is known, however, that the log-returns of most financial assets have an actual kurtosis that is higher than that of the Normal distribution. Empirical evidence has shown that the Normal distribution is a very poor model to fit log-returns of financial asset such as stocks. Several authors have proposed better models. Notably, [2] used the Normal Inverse Gaussian (NIG) to price synthetic collateralized debt obligations (CDO), [3] proposed the Normal Inverse Gaussian (NIG) Levy process, [4] proposed the Hyperbolic models and their generalization, [5] proposed a form of Insurance risk pricing based on a Normal-based distribution operator, [6] proposed distortion operator by Cauchy distribution under a simple transformation to price contingent claims. In this paper, we aim at using the simulation analysis of Cauchy distortion operator under a simple transformation to compare the results of Wang and NIG distortion operators. MATLAB was used in the simulation of the data in Table 1 and the results from Fig. 1 shows that the Cauchy distortion operator recovers the result of NIG which is even a better model than the result of Wang that uses a Normal distribution.

2 The Normal Inverse Gaussian Distribution and Non-Gaussian Black Scholes Contingent Pricing

The NIG distribution is a member of the wider class of generalized hyperbolic distribution. This larger family was introduced in [1]. It is a well-known fact that the returns of most financial assets have semi-heavy tails and the actual kurtosis is higher than that of a normal distribution. It belongs to the infinitely divisible class of distribution which allows for the construction of a non-Gaussian Black-Scholes option pricing theory see also [7]. The NIG is one of the only two sub-classes being closed under convolutions. Its density function is given in [8] as

$$nig(x;\alpha,\beta,\delta,\mu) = \mu \, \frac{\alpha\delta e^{\delta\gamma} \, K_1}{\pi} \, \frac{(\alpha\sqrt{\delta^2+(x-\mu)^2} \,)e^{\beta(x-\mu)}}{\sqrt{\delta^2+(x-\mu)^2}} \qquad x \in \mathcal{R}. \tag{1}$$

Where K_λ is the modified Bessel function of the third kind with index λ given by

$$K_\lambda(x) = \int_0^\infty U^{\lambda-1} \, e^{-\frac{1}{2}x(U^{-1}+\nu)} du\,, \qquad x > 0,$$

and

$$\gamma^2 = \alpha^2 - \beta^2.$$

The parameter domain is $\delta > 0, \ \alpha \geq 0, \ \alpha^2 > \beta^2 \ and \ \mu\epsilon\mathcal{R}$. The parameter $\alpha > 0$ determines the shape, β with $0 \leq |\beta| < \alpha$ the skewness and $\mu\epsilon\mathcal{R}$ the location and $\delta > 0$ is a scaling parameter.

$$NIG(x;\alpha,\beta,\delta,\mu) = \int_{-\infty}^{\alpha} nig(y;\alpha,\beta,\delta,\mu)dy$$

The distribution has the survival function $\overline{NIG}$ given as

$$\overline{NIG}(x;\alpha,\beta,\delta,\mu) = \int_x^\infty nig(y;\alpha,\beta,\delta,\mu)dy.$$

The mean is given as

$$E[X] = \mu + \frac{\delta\beta}{\sqrt{\alpha^2-\beta^2}}.$$

The variance is given as

$$Var\,(t,\tau) = e^{r\tau}V(t,\tau) - \mu^2(t,\tau)$$

$$Skew(t,\tau) = \frac{e^{r\tau}W(t,\tau) - 3\mu(t,\tau)e^{r\tau}V(t,\tau) + 2\mu(t,\tau)^3}{[e^{r\tau}V(r,\tau) - \mu(t,\tau)^2]^{\frac{3}{2}}}$$

$$Kurt(t,\tau) = \frac{e^{r\tau}X(r,\tau) - 4\mu(t,\tau)e^{r\tau}W(r,\tau) + 6e^{r\tau}\mu((t,\tau)^3)\,V(t,\tau) - 3\mu(t,\tau)^4}{[e^{r\tau}V(r,\tau) - \mu(t,\tau)^2]^2}$$

Where

$$V(t,\tau) = \int_{S_t}^{\infty}\frac{2\left(1-ln\left(\frac{k}{S_t}\right)\right)}{K^2}\,C(t,\tau,K)dK\; + \int_0^{S_t}\frac{2\left(1-ln\left(\frac{k}{S_t}\right)\right)}{K^2}\,P(t,\tau,K)dK$$

$$W(t,\tau) = \int_{S_t}^{\infty}\frac{6ln(\frac{K}{S_t})-3\left(\ln\left(\frac{k}{S_t}\right)\right)^2 C(t,\tau,K)dK}{K^2} \;+\; \int_0^{S_t}\frac{6ln(\frac{K}{S_t})-3\left(\ln\left(\frac{k}{S_t}\right)\right)^2 P(t,\tau,K)\,dK}{K^2} \quad ,$$

$$X(t,\tau) = \int_{S_t}^{\infty}\frac{12\ln\,(\frac{K}{S_t})^2)-4(\left(\ln\left(\frac{k}{S_t}\right)\right)^3 C(t,\tau,K)}{K^2}\;dK\; + \int_0^{S_t}\frac{12ln(\frac{K}{S_t})^2)-4\left(\left(ln\frac{k}{S_t}\right)\right)^3 (P(t,\tau,K)}{K^2} \quad ,$$

and

$$\mu(t,\tau) = e^{r\tau} - 1 - e^{r\tau}\,{}^{V(t,r)}\!/_{2} - e^{r\tau}\,W(t,\tau)/6 - e^{r\tau}\;{}^{X(t,\tau)}\!/_{24} \quad .$$

Interestingly, the normal distribution is a limiting case of the NIG distribution. See [9,10,11] for details.

Remark 1.

A Normal distribution with mean μ and variance δ^2 is obtained as a limiting case of the normal inverse Gaussian distribution (NIG) for$\delta \to \infty\; and\, \frac{\delta}{\alpha} \to \sigma^2$.

An interesting feature of the NIG density is that, unlike the normal density, it is not symmetric and that its asymmetry is determined by the parameter β.

The Laplace transform of the NIG distribution [12], is given as

$$\mathcal{L}(Z) = e^{-\mu z+\delta(\gamma-\gamma_Z)}, \qquad |\beta - Z| < \alpha \quad . \tag{2}$$

Where

$$\gamma^2 = \alpha^2 - \beta^2 \text{ and } \gamma_z^2 = \alpha^2 - (\beta - Z)^2.$$

This form of the Laplace transform yields an expression for the expectation of the exponential transformation of a NIG random variable.

$$E[e^x] = e^{\mu-\delta}\left[\sqrt{\alpha^2 - (\beta+1)^2} - \sqrt{\alpha^2 - \beta^2}\,\right].$$

2.1 The Black-Scholes Type Formula for a NIG Distortion Distribution

For a European call with Strike K for a NIG distortion distribution is given as

$$C(t,S_t) = S_t\overline{NIG}\left(ln\tfrac{K}{S_t}; \alpha,\beta+1,\delta(T-t),[\mu+\theta^*](T-t)\right) - Ke^{-r(T-t)}\overline{NIG}\left(ln\tfrac{K}{S_t}; \alpha,\beta,\delta(T-t),[\mu+\theta^*](T-t)\right). \quad (3)$$

Where $\theta^* = r - \mu + \delta\left[\sqrt{\alpha^2-(\beta+1)^2} - \sqrt{\alpha^2-\beta^2}\right]$

This implies that the price of a standard European pay-off evaluated with the pricing kernel associated to the NIG distortion with parameter θ^* is given by

$$e^{-rT}H[f(S_T,K);\, -\theta\,] = S_0\overline{NIG}\left(ln\tfrac{K}{S_0}; \alpha,\beta+1,\delta(T-t),[\mu+\theta^*]T\right) - -Ke^{-rT}\overline{NIG}\left(ln\tfrac{K}{S_t}; \alpha,\beta,\delta T,[\mu+\theta^*]T\right). \quad (4)$$

2.2 Wang Distortion and Option Pricing

Let X be a random variable representing a financial (insurance) risk and let F_x and S_x be its distribution and Survival function respectively. The premium (price) associated with this position is

$$\pi(X) = \int g(S_X(x))dx \quad (5)$$

where g is an increasing differentiable function with $0 < g < 1\ for\ all\ x$.

Moreover, this function is such that $g(0) = 0\ and\ g(1) = 1$. Equation (5) shows that the premium function π can be seen as a corrected mean

$$\pi(X) = \int xg^{\cdot}\big(S_X(x)\big)d\,F_X(x) = E^{\mu}\,[X] \quad (6)$$

where E^{μ} is the expectation under the density measure μ. [5] proposes the following class of distortion function based on the normal distribution in order to price insurance and financial risks.

$$g_\alpha(u) = \Phi(\Phi^{-1}(u) + \alpha\,) \quad (7)$$

where Φ is the standard normal cumulative distribution function. [13] shows that the distribution in (7) is consistent with Black-Scholes formula. Let us consider the following price kernel associated with distortion in (7).

$$H[X = h(Z), \alpha\,] = \int g_\alpha(S_X(x)dx$$

where h is a continuous, positive and increasing function. For a normal random variable Z we have;

$$H[X = h(Z), \alpha\,] = E[h(Z+\alpha)].$$

In the Standard Black-Scholes model, asset prices follow a geometric Brownian motion with

$$\frac{dX(t)}{X(t)} = \mu dt + \sigma d\omega_t$$

So that

$$X_t = X_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\omega_t}.$$

A standard European call option has pay-off at maturity T and we can write $f(Z)$ as, where Z is a standard normal random variable

$$f(Z) = X_0 (e^{(\mu - \frac{\sigma^2}{2})T + \sigma\sqrt{T}\, Z} - K)$$

Applying the relation (kernel), we obtain

$$H(C(T,K);\, -\alpha) = E[f(Z+\alpha)] = \int_{-\infty}^{\infty} X_0 e^{(\mu - \frac{\sigma^2}{2})T + \sigma\sqrt{T}\,\alpha + \sigma\sqrt{T}\, Z} - K)^{+} \frac{1}{\sqrt{2\pi}} e^{-\frac{\sigma^2}{2}} dz.$$

The values of Z for which

$$X_0 e^{(\mu - \frac{\sigma^2}{2})T + \sigma\sqrt{T}\,\alpha + \sigma\sqrt{T}\, Z} \geq K$$

determines the region of integration.

This region is $[Z_{min}, \infty]$ where

$$Z_{min} = \frac{ln\frac{K}{X_0} - (\mu - \frac{\sigma^2}{2})\, T - \sigma\sqrt{T}\,\alpha}{\sigma\sqrt{T}}.$$

Calibrating Wang's discounted certainty equivalent to the underlying price using

$$\alpha = \frac{(\mu - r_{c)}}{\sigma} \sqrt{T}$$

gives

$$e^{r_c T} H(C(T,K), -\alpha) = X_0 \Phi\,(\frac{\ln\frac{X_0}{K} + (r_c + \frac{\sigma^2}{2})\, T}{\sigma\sqrt{T}} - e^{r_c T} K \Phi(\frac{\ln\frac{X_0}{K} + (-\frac{\sigma^2}{2})\, T}{\sigma\sqrt{T}}\,. \quad (8)$$

which is the Black-Scholes price of the call option at time 0.

3 The Cauchy Distortion Operator (under a Simple Transformation) and Option Prices

A transformation variable Y follows a Cauchy distribution under a simple transformation of dividing through a constant with parameter vector (a,b), in symbolic notation $X \sim$ cauchy $(a\,, b)$, if its pdf is given by

$$f(S_{t,}\, a, b\,) = \frac{b\theta}{\pi}\left(\frac{1}{b^2 + (S-a)^2}\right), \qquad -1 < S_t < 1$$

Where θ is the stabilization term and it is given by

$$\theta = \frac{\pi}{2btan^{-1}\left(\frac{1-a}{b}\right)}$$

The pdf can therefore be written as

$$f(S_{t,}a,b) = \frac{1}{2tan^{-1}(\frac{1-a}{b})\,(b^2+(S-a)^2}\qquad ,-1 < S_t < 1\ \ ,a < 1\ \ ,b > 0.$$

The characteristic function of the pdf is given by

$$\varphi(S) = \frac{\theta}{\pi}\int_{-\infty}^{\infty} e^{ibS}\frac{1}{b^2+(S-a)^2}\,ds = \ \frac{\theta}{\pi}\ e^{iaS-bS}.$$

See [14,15], for more details and derivation of Cauchy distribution under a simple transformation of dividing by a constant.

The first four moment of this pdf are found in Appendix 2.

Consider the Cauchy distortion under a transformation defined in [16]. Let Z be the random variable with distribution given by $C\ (a,b)$ and let $X = h(Z)$ be a transformation through a continuous, positive and increasing function h.

We now have to study how this distortion affects an exponential Levy model for assets prices and in particular if there is a value θ such that discounted asset prices behave like risk-neutral prices.

Let us consider the following exponential Cauchy (under transformation) asset price model.

$$X_t = \ X_0\, e^{Z_t}\qquad ,t > 0$$

where Z_t is a $(\Im_t\ ,p\ )$ Cauchy-Levy (under a transformation) process with parameters (a,b). Then the $(\Im_t\ ,p\ )$ - random variable X_t is the price of the security at time T and it can be written as

$$X_T = h(Z_T).$$

For a function $h(U) = \ X_0\ e^U$ and a random variable Z_t with distribution $C(a,b)$, then we have,

$$H[X_T,-\theta] = E[f(Z+\theta)] = \left(E(0)e^{\beta\delta^2 I_t+\sigma\sqrt{T}\,Z-\frac{1}{2}t} - K\right)_+.$$

Now, we need to choose θ such that the discounted price process

$$\{\exp(-(r-q)t)\,X_t,\qquad t \geq 0\}$$

is a martingale i.e

$$X_0 = \exp\,(-(r-q)t\;E^{\theta}[X_t]. \tag{9}$$

Expectation is taken with respect to law with density $f_t^{\theta}(x)\,,q$ is the rate of yield of compound dividends per annum and r the interest rate.

Let

$$\emptyset(U) = E[\exp\,(U_i,X_i]$$

denotes the characteristics function of X_i, then from (9) that in order to let the discounted price process be a martingale, [17,18], we need to have;

$$\exp(r-q) = \ \frac{\emptyset(-i(\theta+1)}{\emptyset(-\theta)}\,. \tag{10}$$

It is easy to see that

$$\theta = \frac{\mu - r + q - 2(a-b)}{2(a-b)} \tag{11}$$

such that

$$X_t = W_t + \frac{\mu - r + q - 2(a-b)}{2(a-b)} \tag{12}$$

and so,

$$\zeta_t \triangleq \frac{d\phi^{\zeta}}{\phi}/\Im_t = exp\{2(a-b)X_t - 2(a-b)_t\}.$$

Now,

$$E_t = E_0 \exp\{2(a-b)X_t - 2(a-b)t - (r-\mu)t\}.$$

The pay off of the contract will be $C_T = E_T - K$ as a function of X_T gives (10) and so using that $\{X_t\}_{t\geq 0}$ is a Q-Brownian motion gives the fair value of K as

$$K = E^Q[E_T] = e^{(r-\mu)}E_0.$$

Hence calibrating Cauchy discounting certainty equivalent to the underlying security price using (11) gives

$$e^{-r_c T}H\big(f(E(T),K)\big) = E(0)C\left(\frac{\ln\left(\frac{X_0}{K}\right)+\mu-r+q+2(a-b)T}{2(a-b)\sqrt{T}}\right) - KCe^{-r_c T}\left(\frac{\ln\left(\frac{X_0}{K}\right)\pm r+q-2(a-b)T}{2(a-b)\sqrt{T}}\right) \tag{13}$$

which is the Black-Scholes price of the call option at time 0.

3.1 The Tables of the Simulated Data of the Wang, NIG and Cauchy Pricing Processes

Table 1 shows the numerical option prices from a sample of asset prices simulated from various model viz: Black-Scholes model, CEV model, Mertons model, NIG and Cauchy models.

3.2 The Graph of the Distortion Functions

The Strike prices of the distortion functions is plotted against the simulated prices of Wang, NIG and Cauchy distortion function respectively.

3.3 Discussion

The data where simulated for different models at various strike prices between 16 and 25 for Wang, NIG and Cauchy distortion functions respectively using MATLAB Program. The graphs were also plotted using MATLAB command for Fig. 1 (see Figs. 2, 3 and 4 are in appendix 1). From the graph in Fig. 1, we discover the behavior of the distortion functions at various strike prices. The NIG and Cauchy distributions, modeled the prices better than the Wang at various strike prices. More so, the similarities of the graphs of NIG and Cauchy distortion functions in Fig. 1 is a clear indication that Cauchy distortion function can recover the results of NIG distortion function.

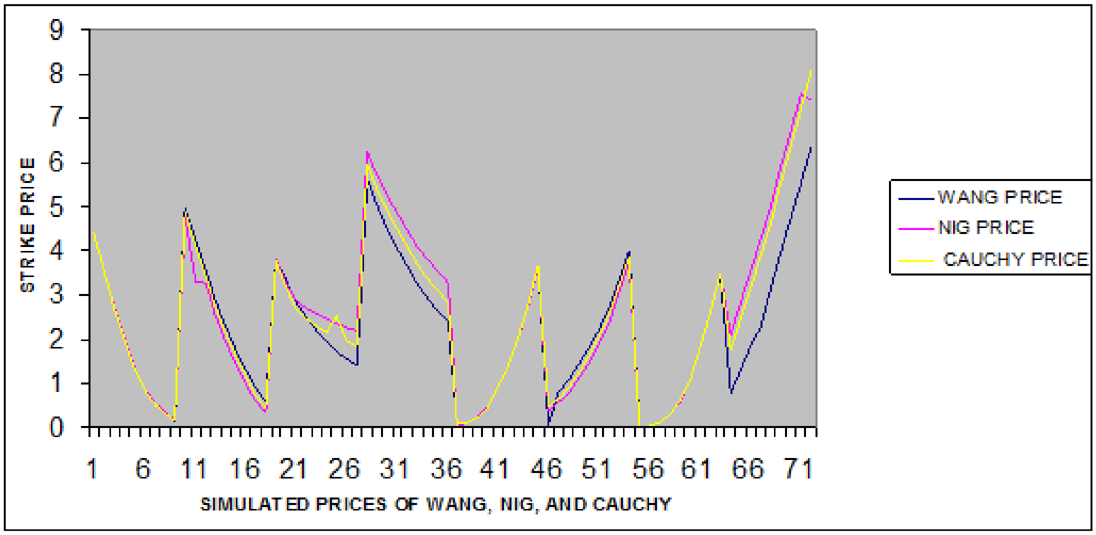

Fig. 1. The graph of Strike price against Wang, NIG and Cauchy Prices

Table 1. Numerical option prices from a sample of asset prices simulated from a Black-Scholes, model, CEV model, Merton's model, NIG model and Cauchy model

Strike price	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Wang price	4.429946	3.520876	2.684585	1.943541	1.335626	0.869027	0.541156	0.320120	01181760
	5.0011123	4.257994	8.55319	2.897782	2.300873	1.760999	1.287738	0.8940492	0.584315
	3.82582	3.30389	2.859597	2.495561	2.190748	1.937706	1.729408	1.554798	1.406543
	5.623651	5.040767	4'516208	4.044822	3.625175	3.253745	2.922809	2.634771	2.384961
	0.052	0.1085	0.2478	0.494	0.8704	1.3917	2.0479	2.8016	3.6284
	0.06029	0.8187	1.0805	1.4004	1.7791	2.2244	2.7341	3.3184	3.9862
	0.0044	0.026	0.098	0.2739	0.5939	1.0898	1.7629	2.5702	3.4596
	0.7663	1.2402	1.8105	2.2474	3.1651	3.9185	4.7137	5.5377	6.381
Nig price	4.429254	3.519726	2.683260	1.942495	1.335341	0.869695	0.542594	0.321952	0.183597
	4.794092	3.29805	3.254972	2.569042	1.954117	1.410764	0.952544	0.595426	0.340217
	3.798796	3.250806	2.882139	2.693118	2.55961	2.45169	2.358794	2.275172	2.198198
	6.241786	5.729932	5.265508	4.843516	4.462434	4.119352	3.80827	3.531309	3.28515
	0.0473	0.1037	0.2438	0.492	0.8711	1.3949	2.0528	2.807	3.6336
	0.3872	0.5599	0.7826	1.0721	1.4328	1.877	2.4033	3.0238	3.7454
	0.0025	0.0225	0.0913	0.2633	0.5806	1.0764	1.752	2.563	3.4557
	2.1017	2.774	3.4934	4.2503	5.0381	5.8509	6.6855	7.5378	7.4021
Cauchy price	4.425713	3.512952	2.672740	1.921475	1.307166	0.838378	0.516315	0.301095	0.169034
	4.871829	4.10258	3.392299	2.73939	2.152196	1.614958	1.136832	0.741379	0.440127
	3.7663106	3.124439	2.682044	2.458136	2.295305	2.160739	2.515896	1.938743	1.841157
	5.982305	5.430645	4.930659	4.476909	4.069693	3.706596	3.3779344	3.0938	2.845882
	0.0538	0.100633	0.237467	0.483733	0.862333	1.3919	2.060967	2.820267	3.646267
	0.45667	0.649333	0.901167	1.223733	1.612367	2.070067	2.5821	3.165933	3.842733
	0.001533	0.022333	0.094476	0.274233	0.593633	1.08267	1.760033	2.568733	3.45667
	1.760033	2.40467	3.108367	3.861667	4.655167	5.479633	6.332367	7.09967	8.0914

4 Conclusion

The Cauchy distribution is well known example of a stable distribution [19]. In fact, the Gaussian and Cauchy distribution are the only two stable distribution for which closed form mathematical formula exist and it is consistent with the behavior we observe in real capital markets. Equation (11) demonstrates that

Cauchy distortion function approach under a transformation recovers the Black-Scholes price of a European Call option with (13). Also, in this paper, we propose a simulation analysis of Cauchy distortion operator via MATLAB to compare the Wang distortion and NIG distortion operator with their pricing model complex. Under the same conditions and in similar situation, the option pricing method of Cauchy distortion operator proposed reproduces the results of pricing methods of Normal Inverse Gaussian (NIG) fully established in the literature and Black-Scholes Model.

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Competing Interests

Authors have declared that no competing interests exist.

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Appendix 1

Table 1. Numerical option prices from a sample of asset prices simulated from a Block-Scholes model

Strike price	16	17	18	19	20	21	22	23	24
Wang price	4.429946	3.520876	2.684585	1.943541	1.335626	0.869027	0.541156	0.320120	01181760
Nig price	4.429254	3.519726	2.683260	1.942495	1.335341	0.869695	0.542594	0.321952	0.183597
Cauchy price	4.425713	3.512952	2.672740	1.921475	1.307166	0.838378	0.516315	0.301095	0.169034

Table 2. Numerical option prices Merton from a sample of asset prices simulated from Merton's model

Strike price	16	17	18	19	20	21	22	23	24
Wang price	5.011123	4.257994	3.553190	2.897782	2.300873	1.760999	1.287738	0.8940492	0.584315
Nig price	4.794092	3.298050	3.254972	2.569042	1.954117	1.410764	0.952544	0.595426	0.340217
Cauchy price	4.871829	4.102558	3.392299	2.73939	2.152196	1.614958	1.136832	0.741379	0.440127

Table 3. Numerical option prices from a sample of asset prices simulated from a CEV model

Strike price	2	3	4	5	6	7	8	9	10
Wang price	3.825820	3.30389	20859597	2.495561	2.190748	1.937706	1.729408	1.554798	1.406543
Nig price	3.798796	3.250806	2.882139	2.693118	2.559610	2.451690	2.358794	2.275172	2.198198
Cauchy price	3.763106	3.124439	2.682044	2.458136	2.295305	2.160739	2.515896	1.938743	1.841157

Table 4. Numerical option prices from a sample of asset prices simulated from a NIG model

Strike price	16	17	18	19	20	21	22	23	24
Wang price	5.623651	5.040767	4.516208	4.044822	3.625175	3.253745	2.922809	2.634771	2.384961
Nig price	6.241786	5.729932	5.265508	4.843516	4.462434	4.119352	3.80827	3.531309	3.285150
Cauchy price	5.982305	5.430645	4.930659	4.476909	4.069693	3.706569	3.379344	3.0938	2.845882

Table 5. Numerical option prices from a sample of asset prices simulated from a Black-Scholes model

Strike price	16	17	18	19	20	21	22	23	24
Wang price	0.0520	0.1085	0.2478	0.4940	0.8704	1.3917	2.0479	2.8016	3.6284
Nig price	0.0473	0.1037	0.2438	0.4920	0.8711	1.3949	2.0528	2.8070	3.6336
Cauchy price	0.0538	0.100633	0.237467	0.483733	0.862333	1.3919	2.060967	2.820267	3.646267

Table 6. Numerical option prices from a sample of asset prices simulated from a CEV model

Strike price	16	17	18	19	20	21	22	23	24
Wang price	0.6029	0.8187	1.0805	1.4004	1.7791	2.2244	2.7341	3.3184	3.9862
Nig price	0.3872	0.5599	0.7826	1.0721	1.4328	1.8770	2.4033	3.0238	3.7454
Cauchy price	0.45667	0.649333	0.901167	1.223733	1.612367	2.070067	2.5821	3.165933	3.842733

Table 7. Numerical option prices from a sample of asset prices simulated from a Black-Scholes model

Strike price	16	17	18	19	20	21	22	23	24
Wang price	0.0044	0.0260	0.0980	0.2739	0.5939	1.0898	1.7629	2.5702	3.4596
NIG price	0.0025	0.0225	0.0913	0.2633	0.5806	1.0764	1.7520	2.5630	3.4557
Cauchy price	0.001533	0.022333	0.094467	0.274233	0.593633	1.08267	1.760033	2.568733	3.45667

Table 8. Numerical option prices from a sample of asset prices simulated from a Black-Scholes model

Strike price	16	17	18	19	20	21	22	23	24
Wang price	0.7663	1.2402	1.8105	2.2474	3.1651	3.9185	4.7137	5.5377	6.3810
NIG price	2.1017	2.7740	3.4934	4.2503	5.0381	5.8509	6.6855	7.5378	8.4021
Cauchy price	1.762433	2.40467	3.108367	3.861667	4.655167	5.479633	6.332367	7.205967	8.0914

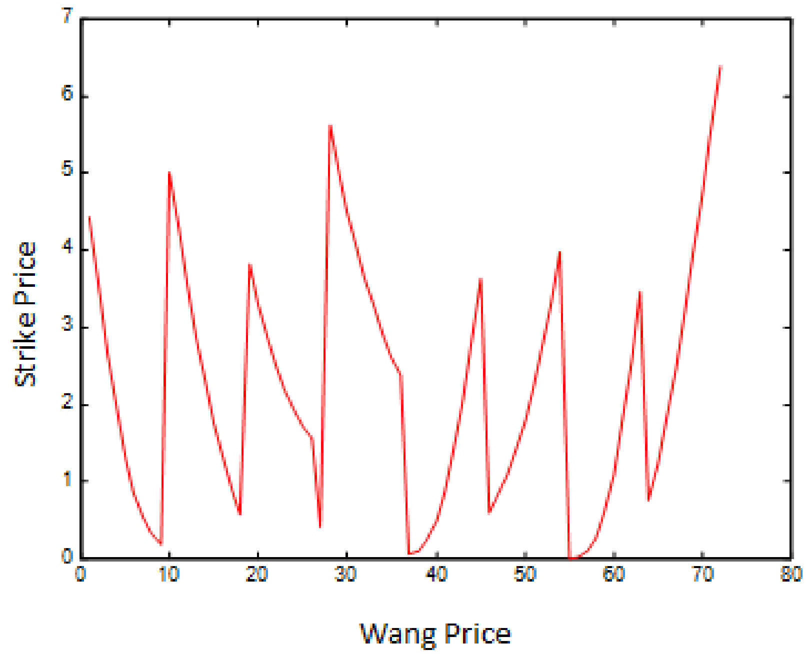

Fig. 1. The graph of Strike Price against Wang Price

The graph of NIG distortion function

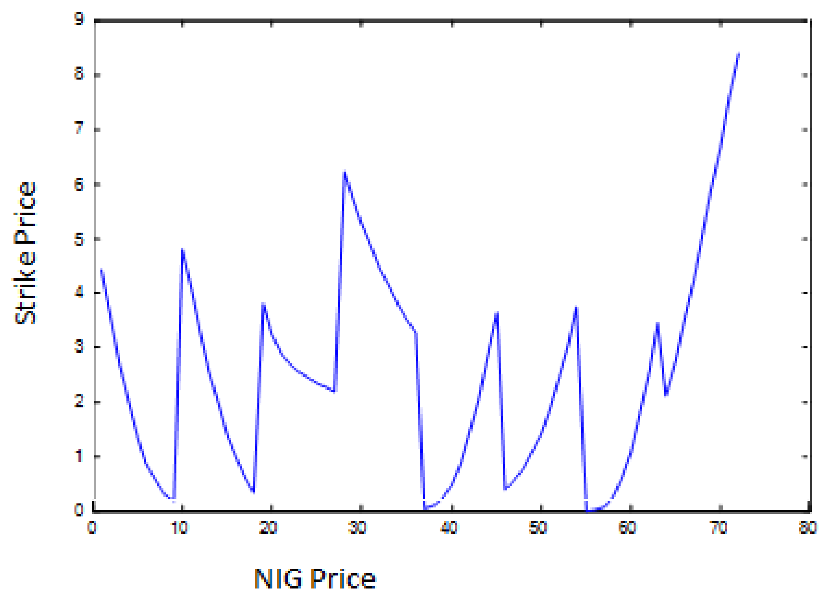

Fig. 2. The graph of strike Price against NIG Price

The graph of Cauchy distortion function

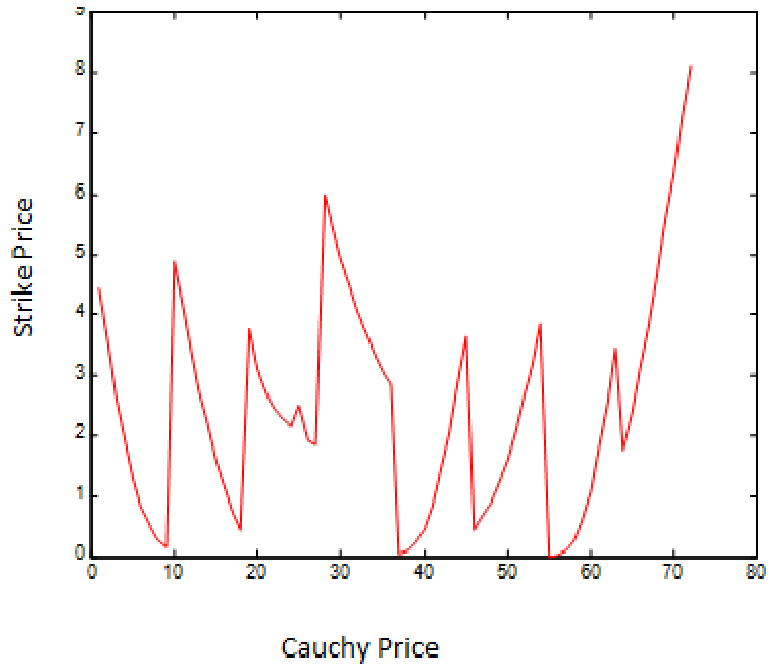

Fig. 3. The graph of Strike Price against Cauchy Price

Appendix 2

$$E(S) = \frac{1}{2tan^{-1}(\frac{1-a}{b})}\left[\frac{1}{2}[\ln(1+a^2+b^2-2a)-\ln(1+a^2+b^2+2a)+\frac{a}{b}[\tan^{-1}(\frac{1-a}{b})-\tan^{-1}(\frac{1-a}{b})\right].$$

$$E(S^2) = \frac{1}{tan^{-1}(\frac{1-a}{b})}\left[1+a[\ln(1+a^2+b^2-2a)-\ln(a^2+b^2)+\frac{a^2-b^2}{b}[tan^{-1}(\frac{1-a}{b})-tan^{-1}(\frac{-1}{b})\right].$$

$$E(S^3) = \frac{1}{2tan^{-1}(\frac{1-a}{b})}\left[4a+\frac{3a^2-b^2}{2}[\ln(1+a^2+b^2-2a)-\ln(1+a^2+b^2+2a) \right.$$
$$\left. +\frac{a^2-3ab^2}{b}[tan^{-1}(\frac{1-a}{b})-tan^{-1}(\frac{-1-a}{b})\right]$$

and

$$E(S^4) = \frac{1}{2tan^{-1}(\frac{1-a}{b})}\left[\frac{1}{3}+a-b^2+3a^2+2a(a^2-b^2)\ln(1+a^2+b^2-2a)+b(b^2-6[tan^{-1}(\frac{\frac{1-b}{b})-2a(a^2-b^2(\ln(a^2-b^2)+b(b^2-6)}{tan^{-1}(\frac{-a}{b})}\right]$$

From the above, we obtain the expression for the skewness and kurtosis as;

$$skew\ (a,b) = \frac{\boldsymbol{\varphi^2(a,b)+\Phi^2(a,b)}}{\boldsymbol{[\varphi^1(a,b)+\Phi^1]^{\frac{3}{2}}}}$$

where

$$\varphi^1(a,b) = 1 = ln[1 + a^2 + b^2 - 2a]^a$$

$$\Phi^1(a,b) = \frac{a^2 - b^2}{b}\left[tan^{-1}\left(\frac{1-a}{b}\right) + tan^{-1}(\tfrac{1-a}{b})\right]$$

$$\varphi^2(a,b) = 4a + \left[\ln\frac{(1 + a^2 + b^2 - 2a)^{\frac{3a^2-b^2}{2}}}{(1 + a^2 + b^2 - 2a)}\right]$$

$$\Phi^2(a,b) = \frac{a^3-3ab^2}{b}[\, tan^{-1}(\tfrac{1-b}{b}) + tan^{-1}(\tfrac{1-b}{b})$$

and

$$Kurt(a,b) = \frac{\varphi^3(a,b) + \Phi^3(a,b)}{\varphi^1(a,b) + \Phi^1(a,b)}$$

with

$$\varphi^3(a,b) = \frac{1}{3} + a - b^2 + 3a^2 + \ln\frac{(1+a^2+b^2-2a)^{\frac{2a(a^2-b^2}{2}}}{a^2+b^2}$$

and

$$\Phi^3(a,b) = b(b^2 - 6)\left[tan^{-1}(\tfrac{1-b}{b}\right) + tan^{-1}(\tfrac{a}{b})].$$

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