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Two-Stage Explicit Stochastic Rational Runge-Kutta Method for Solving Stochastic Ordinary Differential Equations

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Abstract

This paper discussed the derivation of two-stage explicit Stochastic Rational Runge-Kutta (SRRK) methods for the solution of stochastic first order ordinary differential equations. The derivation is based on the use of Taylor series expansion for the deterministic and stochastic parts of the stochastic differential equation. Efforts were made to analyse the stability of the methods and also applied the methods to test some numerical problems to solve Stochastic Differential Equations (SDE). From the results obtained it is obvious that the methods derived performed better than the ones with which we compared our results.

Keywords: Stochastic differential equations; Runge-Kutta methods; explicit rational Runge-Kutta methods.

2010 AMS subject classification: 65L05, 65L06, 65D30.

1 Introduction

Many physical and biological systems are modelled by stochastic differential equations (SDEs), which were obtained by including random effects into the ordinary differential equations. Models of this type offer a more realistic representation of the real physical systems than the deterministic models. However, most of the (SDEs) cannot usually be solved analytically, so numerical methods are needed [1]. Whereas there is a rich theory for designing effective numerical methods for solving ordinary differential equations, the stochastic counterpart are less well developed. Interesting enough, Runge-Kutta methods prove effective in handling stochastic differential equation theories that fits or handle stochastic processes, over some of the analytic methods, or even some numerical schemes [2,3]. Therefore, there is a high need to develop

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stochastic schemes for solving and implementation of Runge-Kutta numerical methods for solving stochastic differential equations [4].

In this paper, two-stage explicit Stochastic Rational Runge-Kutta method is derived based on the modified approach of stochastic Runge-Kutta methods to solve stochastic ordinary differential equation. Consider the non-autonomous, one Wiener SDE of Stratonovich type:

$$dy(t) = f(t, y(t))dt + g(t, y(t)) \circ dW_t \tag{1}$$

The general form of an s-stages explicit Stochastic Rational Runge-Kutta (SRRK) methods is given by [5] as.

$$y_{n+1} = \frac{y_n + h\sum_{i=1}^{s} c_i K_i}{1 + h\sum_{i=1}^{s} y_n v_i H_i} + J_l\left(\sum_{i=1}^{s} S_i Ks_i\right) \tag{2}$$

where $h = \frac{t_{n+1} - t_n}{N}$, N is a positive integer, $J_l = \Delta W_n = \Delta W_{n+1} - \Delta W_n$

$$K_i = f\left(t_n + ha_i, y_n + h\sum_{j=1}^{s} a_{ij} K_j\right) \quad and \quad a_i = \sum_{j=1}^{s} a_{ij}$$

$$H_i = p(t_n, b_i h, y_n + h\sum_{j=1}^{s} b_{ij} H_j) \quad and \quad b_i = \sum_{j=1}^{s} b_{ij}$$

$$p_n(t_n, z_n) = -z_n^2 f(t_n, y_n)$$

and $z_n = 1/y_n$

$$Ks_i = g\left(t_n + has_i, y_n + J_l\sum_{j=1}^{s} bs_{ij} Ks_j\right) \quad and \quad as_i = \sum_{j=1}^{s} bs_{ij}$$

$$\sum_{i=1}^{s} c_i = 1 \quad and \quad \sum_{i=1}^{s} v_i = 1$$

where $c_i, s_i, v_i, a_i, as_i, a_{ij}, b_{ij}, and \quad bs_{ij}, for \quad all \quad i, j = 1,2,..., s$ are constants to be determined.

We can classify SRRK methods, as follows:

If $b_{ij} = a_{ij} = bs_{ij} = 0, \quad \forall i < j,$ then the method is called semi-implicit.

If $b_{ij} = a_{ij} = bs_{ij} = 0, \ \forall i \le j,$ then the method is called explicit.

Otherwise it is called implicit.

2 Derivation of the Methods

In order to derive the two-stage explicit SRRK methods, consider the general form of the explicit SRRK methods, which we shall denote by JAk2.

$$y_{n+1} = \frac{y_n + h\sum_{i=1}^{s} c_i K_i}{1 + h\sum_{i=1}^{s} y_n v_i H_i} + J_l\left(\sum_{i=1}^{s} S_i Ks_i\right) \tag{3}$$

where, h and J_l are as in (3).

When $s = 2$ in (3) we obtain two-stage explicit SRRK given by

$$y_{n+1} = \frac{y_n + h(c_1 K_1 + c_2 K_2)}{1 + hy_n(v_1 H_1 + v_2 H_2)} + J_1(s_1 Ks_1 + s_2 Ks_2) \tag{4}$$

Where

$$K_1 = f(t_n, y_n) \ ; \quad K_2 = f(t_n, a_2 h, y_n + ha_{21}K_1) \ \ and \ \ a_i = \sum_{i}^{2} a_{i,j}$$

$$H_1 = p(t_n, z_n) \ ; \quad H_2 = p(t_n, b_2 h, z_n + hb_{21}H_1) \ \ and \ \ b_i = \sum_{i=1}^{2} b_{i,j}$$

$$Ks_1 = g(t_n, y_n) \ ; \quad Ks_2 = g(t_n + as_2 J_1, y_n + J_1 bs_{21} Ks_1) \ \ and \ \ as_i = \sum_{i=1}^{2} bs_{i,j}$$

$$H_1 = p(t_n, z_n) = -Z^2 f(t_n, y_n), \ \ Z_n = \frac{1}{y_n}$$

$$h = \frac{t_{n+1} - t_n}{N}, N \ \ is \ \ a \ \ Positive \ \ \text{int}\, eger, \ \ J_l = \Delta W_n = \Delta W_{n+1} - \Delta W_n$$

$$and \quad b_2 = b_{21}, a_2 = a_{21}, as_2 = bs_{21}$$

$$\sum_{i=1}^{s} c_i = 1 \ \ and \ \ \sum_{i=1}^{s} v_i = 1$$

are to ensure consistency of the schemes and the constants, $c_1, c_2, v_1, v_2, s_1, s_2, a_2, b_2$ and as_2 are to be determined.

Expanding the RHS of (5) binomially and simplify to get

$$y_{n+1} = [y_n + h(c_1K_1 + c_2K_2)]\big(1 + hy_n(v_1H_1 + v_2H_2)\big)^{-1} + J_1(s_1Ks_1 + s_2Ks_2) \tag{5}$$

$$y_{n+1} = [y_n + h(c_1K_1 + c_2K_2)]\big(1 - (hy_n(v_1H_1 + v_2H_2)\big) + (hy_n(v_1H_1 + v_2H_2))^2 + (hy_n(v_1H_1 + v_2H_2))^3 + \ldots + J_1(s_1Ks_1 + s_2Ks_2) \tag{6}$$

$$y_{n+1} = y_n + h(c_1K_1 + c_2K_2) + \ldots - hy_n(v_1H_1 + v_2H_2) + \ldots + J_1(s_1Ks_1 + s_2Ks_2) \tag{7}$$

Expanding K_2 using Taylor series about, (t_n, y_n) we have:

$$K_2 = f(t_n + a_2h, y_n + ha_{21}K_1) = f + a_2h(f_t + ff_y) + \frac{a_2^2h^2}{2!}(f_{tt} + 2ff_{ty} + f^2 f_{yy}) + \ldots \tag{8}$$

Similarly, expanding H_2 and Ks_2 about (t_n, z_n) and (t_n, y_n) respectively and substituting the expansions in (7) to obtain

$$y_{n+1} = y_n + \left[hc_1 f(t_n, y_n) + c_2hf(t_n, y_n) + c_2a_2h^2(f_t + ff_y) + \frac{c_2a_2^2h^3}{2!}(f_{tt} + 2ff_{ty} + f^2 f_{yy}) + \ldots\right]$$
$$+ [-hy_n^2v_1p(t_n, z_n) - hy_n^2v_2p(t_n, z_n) - h^2y_n^2v_2b_2(p_t + pp_z) - \frac{h^3y_n^2v_2b_2^2}{2!}(p_{tt} + 2pp_{tz} + p^2p_{zz}) + \ldots]$$
$$+ J_1[s_2g + as_2J_1(g_t + gg_y) + \frac{as_2^2J_1^2}{2!}(g_{tt} + 2gg_{ty} + g^2g_{yy}) + \ldots)] \tag{9}$$

Then,

$$y_{n+1} = y_n + hf(c_1 + c_2) + h^2(c_2a_2f_t + c_2a_{21}ff_y) + hp(v_1 + v_2) + h^2(v_2b_2p_t + v_2b_{21}pp_z) + J_1g(s_1 + s_2) + s_2as_2J_1^2(g_t + gg_y) + \ldots \tag{10}$$

But

$$p_n(t_n, z_n) = -z_n^2 f(t_n, y_n) = -Z^2 p(t_n, y_n) \text{ and } z_n = 1/y_n$$

Hence (10) becomes

$$y_{n+1} = y_n + hf(c_1 + c_2) + h^2c_2a_2(f_t + ff_y) + hp(v_1 + v_2) + h^2v_2b_2(p_t + pp_z) + J_1g(s_1 + s_2) + s_2as_2J_1^2(g_t + gg_y) + \ldots \tag{11}$$

We denote solution of the stochastic part by ys and adopt the following notations

$$y' = f(t, y), \quad q' = p(t, y), \quad ys' = g(t, ys) \tag{12}$$

$$y'' = f_t + ff_y \qquad q'' = p_t + pp_y \qquad ys'' = g_t + gg_y \tag{13}$$

$$y''' = f_{tt} + ff_{ty} + f_y(f_t + ff_y) + f(f_{ty} + ff_{yy}) = (f_{tt} + 2ff_{ty} + f^2 f_{yy}) + (f_t f_y + ff_y^2) \tag{14}$$

$$q''' = p_{tt} + pp_{ty} + p_y(p_t + pp_y) + p(p_{ty} + pp_{yy}) = (p_{tt} + 2pp_{ty} + p^2 p_{yy}) + (p_t p_y + pp_y^2) \tag{15}$$

$$ys''' = g_{tt} + gg_{ty} + g_y(g + gg_y) + g(g_{ty} + gg_{yy}) = (g_{tt} + 2gg_{ty} + g^2 g_{yy}) + (g_t g_y + gg_y^2) \tag{16}$$

Substituting (12) – (16) in (11) and truncating after h to the powers two we get

$$y_{n+1} = y_n + hf + \frac{h^2}{2!}(f_t + ff_y) + hp + \frac{h^2}{2!}(p_t + pp_y) + J_1 g + \frac{J_1^2}{2!}(g_t + gg_y) \tag{17}$$

For consistency, we let

$$\begin{aligned} &c_1 + c_2 = 1 \\ &v_1 + v_2 = 1 \\ &c_2 a_2 = v_2 b_2 = \frac{1}{2} \\ &s_1 + s_2 = 1 \\ &s_2 as_2 = \frac{1}{2} \end{aligned} \tag{18}$$

with local truncation error of order h^3, J^3.

Case 1:

If $c_1 = v_1 = s_1 = \frac{1}{4}, \quad c_2 = v_2 = s_2 = \frac{3}{4}, \quad a_2 = b_2 = as_2 = \frac{2}{3}$ we have

$$y_{n+1} = \frac{y_n + h(\frac{1}{4}K_1 + \frac{3}{4}K_2)}{1 + hy_n(\frac{1}{4}H_1 + \frac{3}{4}H_2} + J_1(\frac{1}{4}Ks_1 + \frac{3}{4}Ks_2) \tag{19}$$

where $K_1, H_1, K_2, H_2, Ks_1, Ks_2, h, J_1$ are as defined in (4)

Case 2:

If $c_1 = v_1 = 0, \quad c_2 = v_2 = 1, \quad s_1 = s_2 = \frac{1}{2}, a_2 = b_2 = \frac{1}{2}, as_2 = 1$, then

$$y_{n+1} = \frac{y_n + h(K_2)}{1 + hy_n(H_2)} + J_1(\frac{1}{2}Ks_1 + \frac{1}{2}Ks_2) \quad (20)$$

where $K_1, H_1, K_2, H_2, Ks_1, Ks_2, h, J_1$ are as defined in (4).

Case 3:

If we let $c_1 = c_2 = v_1 = v_2 = \frac{1}{2}, s_1 = \frac{1}{4}, s_2 = \frac{3}{4}, a_2 = b_2 = 1, as_2 = \frac{2}{3}$

Then we obtain

$$y_{n+1} = \frac{y_n + h(\frac{1}{2}K_1 + \frac{1}{2}K_2)}{1 + hy_n(\frac{1}{2}H_1 + \frac{1}{2}H_2)} + J_1(\frac{1}{4}Ks_1 + \frac{3}{4}Ks_2) \quad (21)$$

where $K_1, H_1, K_2, H_2, Ks_1, Ks_2, h, J_1$ are as defined in (7)

3 Stability Analysis of the Two-Stage Schemes

Theorem 1: (Convergence, [6])

(i) Let the function $\phi(x, y, h)$ be continuously jointly as a function of its three arguments, in the region F defined by $X \in [a,b], y \in (-\infty, \infty), h \in [0, h_0] \quad h_0 > 0$

(ii) Let $\phi(h, y, h)$ satisfy a Lipchitz condition of the form $\left|\phi(x, y^*, h) - \phi(x, y, h)\right| \le M\left|y^* - y\right|$ for all points $(x, y^*, h), (x, y, h) \quad in \quad \mathscr{F}$.

Then the method $y_{n+1} - y_n = h\phi(x_n, y_n, h)$ is convergent if and only if it is consistent.

For the stability analysis of the derived schemes, we shall adopt the principles of [8,9,10,11,12]. Since the stability analysis of the deterministic method corresponds with the stability of the corresponding stochastic method [6,7]. Therefore, for the stability of the stochastic methods, it is sufficient to analyse the stability of the corresponding deterministic methods.

From (19) the corresponding deterministic method can be written as

$$y_{n+1} = \left\{ y_n + \left(\frac{hK_1}{4} + \frac{3hK_2}{4} \right) \right\} \left\{ 1 + \left(\frac{hy_nH_1}{4} + \frac{3hy_nH_2}{4} \right)^{-1} \right\} \quad (22)$$

If we expand (22) using binomial expansion, simplify and truncate h after the powers of two we have

$$y_{n+1} = y_n - \frac{hy_n^2H_1}{4} - \frac{3hy_n^2H_2}{4} + \frac{hK_1}{4} + \frac{3hK_2}{4} + \frac{h^2y_n^3H_1}{16} + \frac{3h^2y_n^3H_1}{16} + \frac{3h^2y_n^3H_1H_2}{16} - \frac{9h^2y_n^2K_1H_1}{16} - \frac{3h^2y_nK_1H_2}{16} + \frac{9h^2y_n^3H_2^2}{16} - \frac{3h^2y_nH_1H_2}{16} - \frac{9h^2y_nK_2H_2}{16} \tag{23}$$

Let

$$y' = \lambda y, \quad and \quad y' = f(t_n, y_n) = f_n$$

$$H_1 = p(t_n, z_n) = -z^2 f(t_n, y_n), \quad z_n = \frac{1}{y_n}$$

and $p(t_n, z_n) = -z^2 f(t_n, y_n) = -z^2\lambda y_n$

then,

$$K_1 = \lambda y$$

$$K_2 = \lambda h(1 + \frac{3}{4}\lambda h)y_n$$

$$H_1 = -\frac{1}{y^2}\lambda y_n$$

$$H_2 = -\frac{1}{y^2}\lambda h(1 + \frac{3}{4}\lambda h)y_n$$

Substituting these in (24), we have

$$y_{n+1} = y_n - \frac{hy_n^2\left(-\frac{1}{y_n^2}\lambda y_n\right)}{4} - \frac{3hy_n^2\frac{\lambda h}{y_n^2}\left(1+\frac{3}{4}\lambda h\right)y_n}{4} + \frac{h\lambda y_n}{4} + \frac{3h\lambda h\left(1+\frac{3}{4}\lambda h\right)y_n}{4} + \frac{h^2y_n^3\left(-\frac{1}{y_n^2}\lambda y_n\right)}{16} + \frac{3h^2y_n^3\left(-\frac{1}{y_n^2}\lambda h\right)}{16} + \frac{3h^2y_n^3\left(-\frac{1}{y_n^2}\lambda y_n\right)\left(-\frac{\lambda h}{y_n^2}\right)\left(1+\frac{3}{4}\lambda h\right)y_n}{16} - \frac{9h^2y_n^2\lambda y_n\left(-\frac{1}{y_n^2}\lambda y_n\right)}{16} - \frac{3h^2y_n\lambda y_n\left(-\frac{1}{y_n^2}\right)\left(1+\frac{3}{4}\lambda h\right)y_n}{16} + \frac{9h^2y_n^3\left\{\left(-\frac{\lambda h}{y_n}\right)\left(1+\frac{3}{4}\lambda h\right)y_n\right\}^2}{16} - \frac{3h^2y_n\left(-\frac{\lambda y_n}{y_n^2}\right)\lambda h\left(1+\frac{3}{4}\lambda h\right)y_n}{16} - \frac{9h^2y_n\lambda h\left(1+\frac{3}{4}\lambda h\right)y_n\left(-\frac{\lambda h}{y_n^2}\right)\left(1+\frac{3}{4}\lambda h\right)y_n}{16}$$

Truncating terms in h of power three and higher, simplify and rearrange the expression in ascending powers of λh, we get

$$y_{n+1} = y_n + \frac{\lambda h y_n}{2} + \frac{9h^2\lambda^2 y_n}{16} \tag{24}$$

$y_{n+1} = \left(1 + \frac{\lambda h}{2} + \frac{9(\lambda h)^2}{16}\right) y_n$, which is a first order difference equation.

Let

$$\xi = 1 + \frac{\lambda h}{2} + \frac{9(\lambda h)^2}{16} \tag{25}$$

be the characteristic equation, for the absolute stability region, we require $|\xi| \leq 1$ where $\lambda < 0$ therefore,

$$-1 \leq 1 + \frac{\lambda h}{2} + \frac{(3\lambda h)^2}{16} \leq 1$$

Hence the interval of the absolute stability of the two-stage (20) is (-4.44, 0).

The interval of absolute stability of the two-stage (20) is (-3, 0) while that of (22) is (-12, 0).

4 Numerical Examples and Results

Problem 1

Consider the SDE [3].

$$dy(t) = -\frac{1}{4} y(1 - y^2)dt + \frac{1}{2}(1 - y^2)dW(t), \quad t \in [0,1] \quad y(0) = 0$$

with the exact solution given by:

$$y(t) = \tanh(0.5)W(t) + \tanh^{-1}(y(0))$$

Problem 2:

Consider the SDE [13].

$$dy(t) = -(\alpha + \beta^2 y)(1 - y^2)dt + \beta(1 - y^2)dW \quad y(0) = y_0 \quad t \in [0,1]$$

with initial condition $y(0) = 0$, where the exact solution is given by:

$$y(t) = \frac{(1 + y(0))e^{(-2t + 0.02W(t))} + y(0) - 1}{(1 + y(0))e^{(-2t + 0.02W(t))} - y(0) + 1}, \quad \alpha = 1, \quad \beta = 0.01, \quad \varepsilon = 0.001, \quad N = 500$$

Therefore, the numerical solution of the explicit SRRK methods for the two-stage schemes as obtained in this work with absolute errors are given in the Tables 1 and 2. The following notations will be used to represents results in the tables below RAe1-3: Results obtained by [3] Logmani: Results obtained by [13] JAk 2: Results obtained by our new methods.

Table 1. Numerical results of two-stage JAk 2 explicit SRRK in comparison with [3] for Problem 1

t_i	W_i	**Exact solution**	**PL**	**Absolute error**	**RAe1 (Pa)**	**Absolute error**	**RAe2**	**Absolute error**	**RAE3**	**Absolute error**	**JAk 2**	**Absolute error**
0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	-0.0439	-0.0219	-0.0219	0	-0.0219	0	-0,0219	0	-0.0219	0	-0.0219	0.0000
0.2	-0.0679	-0.034	-0.0334	0.0006	-0.0334	0.0005	-0.0334	0.0005	-0.0334	0.0005	-0.034	0.0000
0.3	-0.0473	-0.0237	-0.0223	0.0014	-0.0223	0.0014	-0.0223	0.0014	-0.0223	0.0014	-0.0237	0.0000
0.4	-0.0951	-0.0475	-0.0456	0.0019	-0.0456	0.0019	-0.0456	0.0019	-0.0456	0.0019	-0.0475	0.0000
0.5	-0.1686	-0.0841	-0.081	0.0031	-0.0811	0.003	-0.081	0.003	-0.0811	0.003	-0.0843	0.0002
0.6	0.0044	0.0022	0.0072	0.0005	0.0072	0.005	0.0071	0.0049	0.0072	0.005	0.0022	0.0000
0.7	-0.0121	-0.006	-0.0012	0.0048	-0.0012	0.0049	-0.0013	0.0048	-0.0012	0.0049	-0.006	0.0000
0.8	0.0556	0.0278	0.0327	0.0048	0.0327	0.0049	0.0326	0.0048	0.0327	0.0049	0.0278	0.0000
0.9	0.2192	0.1092	0.113	0.0039	0.1132	0.004	0.113	0.0038	0.1132	0.004	0.1096	0.0004
1.0	0.0809	0.0404	0.0416	0.0012	0.0417	0.0013	0.0416	0.0012	0.0417	0.0013	0.0405	0.0000

Table 2. Numerical results of two-stage JAk 2 explicit SRRK in comparison with [13] for Problem 2

	PL	**R2**	**SIM**	**IM**	**SIM3**	**JAK 2**
h	Error	Error	Error	Error	Error	Error
0.040	0.007381	0.000111	0.000007	0.000003	0.000000	0.000011
0.020	0.003666	0.000027	0.000001	0.000000	0.000000	0.000000
0.010	0.001827	0.000007	0.000000	0.000000	0.000000	0.000000
0.005	0.000912	0.000001	0.000000	.0.000000	0. 000000	0.000000

5 Discussion of Results

With the derived two-stage explicit Stochastic Rational Runge-Kutta schemes (SRRK) denoted JAk in the numerical results tables. Some of the family schemes were tested on the numerical Problems 1 from [3] and problem 2 from [13]. Matlab software (version 2010) was employed to run the simulations, based on normal distributed random numbers with mean zero and variance (standard deviation) one, i.e N(0,1). From Tables 1and 2, we can see the performance of our family of two-stage schemes with the existing schemes, [3] and [13]. Also detail analysis of each family of the two-stages developed were carried out using Schur method, in line with what we call mean and mean square stability principles in stochastic stability analysis discussed by some authors in section 3.0, of which the stability analysis of each family of two- stage are bounded by the intervals (-4.44), (-3.0,0) and (-12.0, 0) respectively, which are better than counter part deterministic explicit Runge-Kutta methods.

6 Conclusions

Clearly family of two-stage schemes performs better in terms of convergence and accuracy, therefore the SRRK schemes are alternative methods to solve this class of these problems.

Competing Interests

Authors have declared that no competing interests exist.

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