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Modeling the Impact of Optimal Control Strategies on the Dynamics of Zika Virus Disease Using the Sterile Insect Technology

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Authors' contributions

This work was carried out in collaboration among all authors. Author AW designed the study, performed the mathematical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors AJ and DMA managed the model simulation. Author OKS managed the literature searches. Author CEMG supervised the work. All authors read and approved the final manuscript.

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Abstract

This work is aimed at formulating a mathematical model for the control of zika virus infection using Sterile Insect Technology (SIT). The model is extended to incorporate optimal control strategy by introducing three control measures. The optimal control is aimed at minimizing the number of Exposed human, Infected human and the total number of Mosquitoes in a population and as such reducing contacts between mosquitoes and human, human to human and above all, eliminates the population of Mosquitoes. The Pontryagin's maximum principle was used to obtain the necessary conditions, find the optimality system of our model and to obtain solution to the control problem. Numerical simulations result shows that; reduction in the number of Exposed human population, Infected human population and reduction in the entire population of Mosquito population is best achieved using the optimal control strategy.

Keywords: Zika; modeling; virus; wild; sterile; technology.

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1 Introduction

Zika is a viral disease that invades human population by the bite of infected mosquitoes. It was discovered in 1947 in Uganda (zika virus fact sheet, Ethiopian Midwives Association).The disease is commonly contacted through the bites of an infected mosquitoes. Two species of mosquitoes spread the virus to people; the yellow fever mosquitoes (*Aedes aegypti*) and the Asian tiger mosquitoes (*Aedes albopictus*). Both are native to Texas [1]. During the period of 1960-1980 human infections, typically accompanied by mild illness, were found across Africa and Asia. The first large outbreak of disease causing zika infection occurred in the island of Yap, federated states of Micronesia in 2007, indicating that the virus had moved from south-east Asia across the pacific [2]. Zika belongs to the flavivirus family and it is transmitted through daytime-active Aedes mosquitoes, such as *A. aegypti* and *A. albopictus* [3]. It was not considered a relevant pathogen until another large outbreak occurred in Brazil (2015) which revealed that zika infection is associated with fetal microcephaly and Guillain-Barre syndrome, this prompted the world health organization [4], 2017 to declare zika virus a public health emergency of international concern on February 1st, 2016 [1]. The recent zika outbreak in Brazil with over 1.5million estimated cases from 2015-2016 received significant attention globally. The main reasons are its large number of infections, rapid transmission and the increasing rate of reported microcephaly coincided with the infection. The incidence became public health emergency and followed by the warning announcement from the world health organization [4]. Among symptomatic patients, the most common symptoms include popular rash, fever, typically low grade arthralgia, fatigue, non-purulent conjunctivitis myalgia and headache. While other symptoms like retro-orbital pain, oadema, vomiting, sore throat, uveitis and lymphadenopathy are less frequent [1]. A typical feature of zika virus infection is the popular rash that is often pruriginous and starts on the face and or trunk and then spreads throughout the body but may be focal and fugacious [5,6,7].

This work is aimed at formulating a mathematical model to study the impact of optimal control strategy on the transmission dynamics of zika virus disease using the Sterile Insect Technology.

In this study, we extend the work of [8] by incorporating human population into the model for the control of zika virus vector population using the Sterile Insect Technology, where we divide the vector population into the Aquatic class (Eggs, Larva, Pupae), while we divide the Non-Aquatic mosquitoes class into the Male Mosquitoes (M_M), Female Mosquitoes (F_M),Female Non-Sterile Mosquitoes, (F_{NM}), Sterile Male Mosquitoes $\left(M_S\right)$, Females Sterile Mosquitoes (F_{SM}), Female Infected Non-Sterile Mosquitoes (F_{INM}), and Female Infected Sterile Mosquitoes (F_{ISM}). The work of [9] presented a mathematical model for zika virus cross infection between mosquitoes and human. [10] formulated a mathematical model for the transmission dynamics of zika virus infection with combined vaccination and treatment intervention. The work of [11] presented a mathematical model of sterile insect technique for control of anopheles' mosquitoes; their work presents sterile insect technology (SIT) as a non-polluting method for the control of the invading insects that transmit diseases. [12] formulated a mathematical model on the prevention and control of zika as a mosquito-borne and sexually transmitted disease. [13] presented a mathematical model, analysis and simulation of the spread of zika with influence of sexual transmission and preventive measures. [14] formulated a deterministic model for the transmission dynamic of zika that takes into account the aquatic and non-aquatic stages of development.

Optimal control is a vital mathematical method deciding a strategy regarding epidemic control with provided scenarios, [15,16,17,18]. Chaikham N et al. [19] presented a model on the optimal control of zika infection; they used the Pontryagin's maximum principle to determine the necessary conditions for the optimal control. Their result shows that optimal control helps in decreasing the number of individuals infected and at the long run the spread of the disease. Athithan S et al. [20] presented a research work on the Stability Analysis and the Optimal Control of Malaria Model, numerical simulation was performed to see the effect of the control on the disease dynamics, result shows that the optimal control model is sufficient to eradicate the malaria disease. Chatterjee AN et al. [21] presented a research work on the effect of an antiviral drug treatment along

immune activator IL-2: A control-based mathematical approach for HIV infection. The work of [22] also demonstrated a mathematical model on the CTL mediated control of HIV infection in a long term drug therapy. Roy PT et al. [23] presented an optimal control theoretical approach of the effect of HAART on CTL mediated immune cells. There is a possibility that zika virus outbreak may emerge again in the future, especially in the high vector concentrations areas [24], to this end, we present in this work an environmental pleasant control method called the Sterile Insect Technology (SIT) to study the transmission dynamics of zika virus infection where we extended the work of [8] to formulate a mathematical model of zika virus infection using the Sterile Insect Technology with human population incorporated. We also incorporated Optimal Control measures following the method used by [19]. The Sterile Insect Technology is a kind of control for insects like mosquitoes that does not have any harmful effect on the environment. The technique requires the mass rearing and sterilization of the target insects (Male Mosquitoes) with irradiation like the x-rays and the gamma rays which are then released to an endemic area where the compete and mate with the wild female mosquitoes which results to laying of eggs that do not and such they will be fewer wild mosquitoes in the next generation thereby eliminating insect borne diseases, like the zika virus [8].

2 Model Formulation and Procedures

We divide the mosquito life cycle into two stages, the Aquatic and Non-Aquatic class. The Aquatic class is denoted by a single compartment (A).

The Non-Aquatic class is divided into seven Compartments consisting of the Male Mosquitoes (M_M), Female Mosquitoes not due to lay eggs (F_M), Female Non-Sterile Mosquitoes, (F_{NM}), Sterile Male Mosquitoes (M_S), Female Sterile Mosquitoes (F_{SM}), Female Infected Non-Sterile Mosquitoes (F_{INM}), and Female Infected Sterile Mosquitoes (F_{ISM}).

The human population comprises of Susceptible Human (S_H), Exposed Human (E_H), Infected Human (I_H), Infected but on treatment class (I_{HT}) and Recovered Human (R_H).

The Aquatic stage of the mosquitoes, which consists of Eggs, Larva and Pupae population, increases from the oviposition rate of the reproductive mosquitoes. It reduces due to natural death of the Mosquitoes at the rate of (μ_A) and by density dependence death rate of (μ_ρ).

The Female Mosquitoes (F_M) is recruited at the rate of $(A\phi\gamma)$,where (γ) is the maturity rate of Aquatic Mosquitoes to Adult Mosquitoes, (ϕ) is the proportion of emerging females, it is reduced by the mating rate at the level of (β) for Female Mosquitoes to be with Wild Male Mosquitoes or Sterile Male Mosquitoes. The population is reduced finally by death induced due to attempt to seeking for blood meals at the rate of (∂_M) and by natural death at the rate of (μ_F).

The Male Mosquitoes (M_M) is recruited by the proportion of the emerging Male Mosquitoes $(1-\phi)$ that mature to Adult Mosquitoes at the rate (γ), which also reduces by natural death (μ_M).

The Female Non Sterile Mosquitoes (F_{NM}) population is increased by the female Mosquitoes probability to mate with the wild male Mosquitoes which is given by the rate $\frac{M_M}{M_M + M_S}$. This population is reduced by

(ω), the rate by which the Female Non-Sterile Mosquitoes (F_{NM}) are infected and moved to the Females Infected non-Sterile class.

The population is reduced finally by death induced due to attempt at seeking for blood meals at the rate of (∂_M) and by natural death at the rate of (μ_F).

The Female Sterile Mosquitoes (F_{SM}) population is increased by the wild Female Mosquitoes probability to mate with the sterile mosquitoes, which is given by the rate $\frac{M_S}{M_M + M_S}$. The class reduces by (ω), the rate at which (F_{SM}) becomes infected and moves to (F_{ISM}) class. The population is reduced finally by death induced due to attempt at seeking for blood meals at the rate of (∂_M) and by natural death at the rate of (μ_F).

The population of Female Infected Non Sterile Mosquitoes (F_{INM}) is recruited at the rate at which the Female Sterile Mosquitoes (F_{SM}) are infected at the rate of (ω). The population is reduced finally by death induced due to attempt to seeking for blood meals at the rate of (∂_M) and by natural death at the rate of (μ_F).

The (M_S) which denotes the Sterile Male Mosquitoes are released into the population at the level (Λ_2) .However due to some ecological factors that may affect the associating of the wild and the sterile mosquitoes which may include the mosquito breeding location site, we now assume that only a proportion (p) of the released mosquitoes will join wild mosquitoes' population.

Also due to the variations in composition of wild and sterile mosquitoes, a parameter (q) is used to capture the average mating effectiveness of sterile mosquitoes, so that the actual number of Sterile male Mosquitoes competing with wild Mosquitoes is (pqM_S), and as such, the available injected (M_S) into the wild population of mosquitoes that can effectively mate with wild female mosquitoes is $(pq\Lambda_2)$. The population reduces by natural death at the rate of (μ_S).

The Susceptible Human population is recruited at the level of (Λ_3) of which a fraction (ℓ) of those infected at birth joined the Infectious Human population. The population reduces by the rate at which infectious mosquitoes [Female Infected Non-Sterile Mosquitoes (F_{INM}) or Female Infected Sterile Mosquitoes (F_{ISM}) infects Susceptible Human at the levels of (α_1) and (α_2) respectively. In addition, it reduces by the rate at which the Infectious Human [Infected class (I_H), Recovered class (R_H), or Infected but on treatment class (I_{HT}) infects Susceptible Human through sex at the level of (α_3) or (α_4), (α_5) respectively. This is in line with the clinical studies that high viral load was found in the semen and saliva of recovered patients' weeks after recovery [4,14], which means, zika can be transmitted sexually. The population finally reduces by natural death at the rate of (μ_H).

The population of the Exposed Human is generated by infection of Susceptible Individuals at the rate (α). This population reduces by natural death at the of rate (μ_H) and by the rate at which the exposed are finally infectious at the rate of (σ).

The Infected Human class is generated by the incoming of Infected Babies from Infected Mothers at the rate of $(\ell\Lambda_3)$, due to vertical transmission in addition, the population increases at the rate by which the Exposed becomes infected at the level (σ). The class reduces at the rate (θ) by which the infected are taken for treatment and by natural recovery rate of (τ_1). This class reduces finally by both natural and disease induced death rates at the level of (μ_H) and (∂_1) respectively.

The Infected but on treatment class (I_{HT}) is recruited by the incoming of the infected who are taken for treatment at the rate of (θ), this class reduces at the rate by which the infected but on treatment class recovers due to supportive treatment at the rate of (τ_2). It reduces finally by natural death and disease induced death at the levels of (μ_H) and (∂_2) respectively. (∂_2) is assumed to be less than (∂_1).

The Recovered human is recruited at the rate by which the Infected Human recovers naturally at the rate of (τ_1) or due to supportive treatment at the rate of (τ_2). The population reduces by natural death at the rate of (μ_H). The model flow diagram that incorporates this description is shown in Fig. 1.

2.1 Model flow diagram

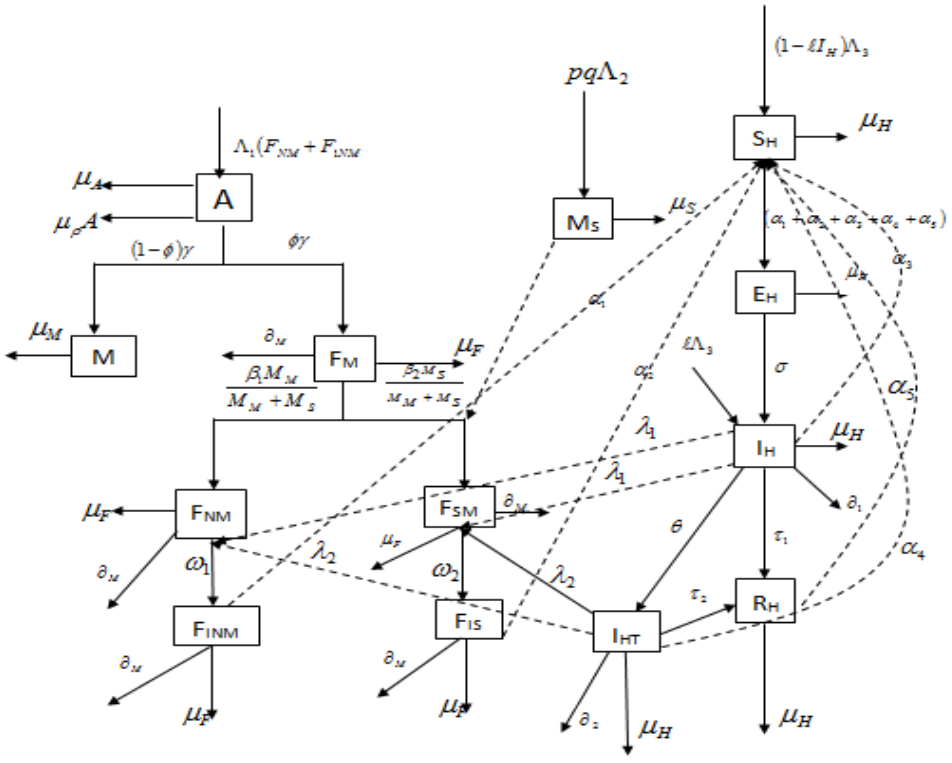

Fig 1. Flow diagram illustrating the interactions of different compartments

Where $\alpha = \frac{\alpha_1 F_{INM} + \alpha_2 F_{ISM} + \alpha_3 I_H + \alpha_4 I_{HT} + \alpha_5 R_H)}{N_H}$. and $\omega = \frac{\lambda_1 I_H + \lambda_2 I_{HT}}{N_H}$

$\omega = \omega_1 = \omega_2$

2.2 Model variables/parameters

2.2.1 Model variables

Descriptions of the model variables used are presented in Table 1.

Table 1. Model variables and descriptions

S/N	Variables	Descriptions
1	A	Aquatic class of Mosquitoes
2	M_M	Male Mosquitoes
3	F_M	Female Mosquitoes not yet laying eggs
4	F_{NM}	Female Non Sterile Mosquitoes
5	M_S	Sterile male mosquitoes
6	F_{SM}	Female Sterile Mosquitoes
7	F_{INM}	Female Infected non-Sterile Mosquitoes
8	F_{ISM}	Female Infected Sterile Mosquitoes
9	S_H	Susceptible Human
10	E_H	Exposed Human
11	I_H	Infected Human
12	I_{HT}	Infected but on treatment human
13	R_H	Recovered Human

2.2.2 Model parameters

Descriptions of the model parameters used is presented in Table 2.

Table 2. Model parameters and descriptions

S/N	Parameters	Descriptions
1	Λ_1	Oviposition rate of Fertilized Female Mosquitoes)
2	ϕ	Proportion of emerging Female Mosquitoes
3	$1-\phi$	Male Mosquitoes emerging population
4	β_i	Mating rate, where $i = 1,2.$
5	γ	Maturity rate of Mosquitoes
6	μ_M	Natural death rate of wild Male Mosquitoes
7	μ_S	Natural death rate of Sterile Mosquitoes

8	μ_ρ	Density dependent death rate of the Aquatic Mosquitoes class
9	μ_H	Natural death rate of Human
10	μ_A	Natural death rate for Aquatic mosquitoes
11	μ_F	Natural death rate for female mosquitoes
12	∂_M	Death induced rate due to attempt by female mosquitoes seeking for blood meals
13	$\frac{M_M}{M_M + M_S} = \rho_W$	Female Mosquitoes probability to mate with wild male Mosquitoes
14	$\frac{M_S}{M_M + M_S} = \rho_S$	Female Mosquitoes probability to mate with Sterilized male Mosquitoes
15	∂_1	Disease Induced death rate for Infected class
16	∂_2	Disease induced death rate for Infected but on treatment class
17	τ_1	Natural recovery rate for human class
18	θ	Rate at which the infected human are taken for treatment
19	τ_2	Recovery rate of the infected but on treatment due to supportive treatment
20	ℓ	Fraction of infected at birth that joined the Susceptible class
21	σ	Rate at which the Exposed becomes infectious
22	Λ_3	Recruitment level into the susceptible human population
23	p	Fraction of the released Sterile Mosquitoes, that can join the wild Mosquitoes,
24	q	Mean mating competitiveness of the sterile male Mosquitoes
25	α	Force of infection for human population
26	α_1	Rate at which Female Infected Non-Sterile Mosquitoes (F_{INM}) infects Susceptible human
27	α_2	Rate at which Female Infected Sterile Mosquitoes (F_{ISM}) infects Susceptible Humans
28	α_3	Rate at which the Infected Human infects Susceptible Human through sex.
29	α_4	Rate at which the Recovered Human infects Susceptible Human through sex.
30	α_5	Rate at which the infected but on treatment human infects Susceptible Human through sex.
31	ω	Force of infection for mosquitoes population
32	ω_1	Rate at which the F_{NM} are infected after biting an infectious human
33	ω_2	Rate at which F_{SM} are infected after biting an infectious human
34	λ_1	Rate at which the Infected human infects susceptible mosquitoes(F_{SM} & F_{NM})
35	λ_2	Rate at which the Infected but treated human infects susceptible mosquitoes(F_{SM} & F_{NM})

2.3 Mathematical model

$$\frac{dA}{dt} = \Lambda_1(F_{NM} + F_{INM}) - \gamma A - \mu_A A - \mu_\rho A^2$$
$$\frac{dF_M}{dt} = \phi\gamma A - [\beta_1\rho_\omega + \beta_2\rho_S]F_M - \partial_M F_M - \mu_F F_M$$
$$\frac{dM_M}{dt} = (1-\phi)\gamma A - \mu_M M_M$$
$$\frac{dF_{NM}}{dt} = \beta_1\rho_\omega F_M - \omega F_{NM} - \partial_M F_{NM} - \mu_F F_{NM}$$
$$\frac{dF_{SM}}{dt} = \beta_2\rho_S F_M - \omega F_{SM} - \partial_M F_{SM} - \mu_F F_{SM}$$
$$\frac{dF_{INM}}{dt} = \omega F_{NM} - \partial_M F_{INM} - \mu_F F_{INM}$$
$$\frac{dF_{ISM}}{dt} = \omega F_{SM} - \partial_M F_{ISM} - \mu_F F_{ISM}$$
$$\frac{dM_S}{dt} = pq\Lambda_2 - \mu_S M_S$$
$$\frac{dS_H}{dt} = (1-\ell)\Lambda_3 - \alpha S_H - \mu_H S_H$$
$$\frac{dE_H}{dt} = \alpha S_H - \sigma E_H - \mu_H E_H$$
$$\frac{dI_H}{dt} = \ell\Lambda_3 + \sigma E_H - \tau_1 I_H - \theta I_H - \partial I_H - \mu_H I_H$$
$$\frac{dI_{HT}}{dt} = \theta I_H - \tau_2 I_{HT} - \partial_2 I_{HT} - \mu_H I_{HT}$$
$$\frac{dR_H}{dt} = \tau_1 I_H - \tau_2 I_{HT} - \mu_H R_H \qquad (2.1)$$

Where $\alpha = \frac{\alpha_1 F_{INM} + \alpha_2 F_{ISM} + \alpha_3 I_H + \alpha_4 I_{HT} + \alpha_5 R_H)}{N_H}$. $\omega = \frac{\lambda_1 I_H + \lambda_2 I_{HT}}{N_H}$

$\frac{M_S}{M_M + M_S} = \rho_S$, $\frac{M_M}{M_M + M_S} = \rho_\omega$ and $\omega_1 = \omega_2 = \omega$

The released sterile male mosquitoes' population can be decoupled from the system (2.1). Since it is independent of other compartments, the size of its population is controlled by human intervention, and as such, it is independent from the rest of the population [8,11,14].

3 Optimal Control Problem

Optimal control deals with the problem of finding a control law for a given system such that a certain optimality principle is obtained. It is a set of differential equations describing the paths of the control variables that minimize the cost function.

The model (2.1) is extended to incorporate optimal control strategy by introducing three control measures. The control variable $u_1(t)$ represents the use of preventive measures such as insect repellent or mosquito net to reduce the contacts between human and mosquito. The control variable $u_2(t)$ represents the measure of abstaining from sexual activities and control variable $u_3(t)$ is the use of pesticides at the mosquito breeding sites to eliminate or reduce the total number of mosquitoes. This control strategy is aimed at reducing the zika virus infection by reducing contacts between mosquitoes and human, human to human and vector elimination.

Consequently, the forces of infection in the human population are reduced by the factors $(1-u_1(t))$ and $(1-u_2(t))$. The force of infection in the vector population is reduced by a factor of $(1-u_1(t))$. The vector birth rate is reduced by a factor of $(1-u_3(t))$ as the death rate of the mosquito population increases by $\zeta u_3(t)$, where $\zeta > 0$, then the dynamic of the system (2.1) with optimal control strategies is governed by the system of differential equations as follows:-

$$
\left.
\begin{aligned}
\frac{dA}{dt} &= \Lambda_1(f_{NM}+f_{INM})(1-u_3)-\gamma A(1-u_3)-(\mu A+\zeta u_3)A-(\mu_\rho+\zeta u_3)A^2\\
\frac{dF_M}{dt} &= \phi\gamma A(1-u_3)-[\beta_1\rho_\omega+\beta_2\rho_s]F_M(1-u_3)-(\partial_M+\zeta u_3)F_M-(\mu_F+\zeta u_3)F_M\\
\frac{dM_M}{dt} &= (1-\phi)\gamma(1-u_3)-(\mu_M+\zeta u_M)M_M\\
\frac{dF_{NM}}{dt} &= \beta_1\rho_\omega F_m-\left(\frac{\lambda_1 I_H+\lambda_2 I_{HT}}{N_H}\right)(1-u_1)F_{NM}-(\partial_M+\zeta u_3)F_{NM}-(\mu_F+\zeta u_3)F_{NM}\\
\frac{dF_{SM}}{dt} &= \beta_2\rho_S F_M-\left(\frac{\lambda_1 I_H+\lambda_2 I_{HT}}{N_H}\right)(1-u_1)F_{SM}-(\partial_M+\zeta u_3)F_{SM}-(\mu_F+\zeta u_3)F_{SM}\\
\frac{dF_{INM}}{dt} &= \left(\frac{\lambda_1 I_H+\lambda_2 I_{HT}}{N_H}\right)(1-u_1)F_{NM}-(\partial_M+\zeta u_3)F_{INM}-(\mu_F+\zeta u_3)F_{INM}\\
\frac{dF_{ISM}}{dt} &= \left(\frac{\lambda_1 I_H+\lambda_2 I_{HT}}{N_H}\right)(1-u_1)F_{SM}-(\partial_M+\zeta u_3)F_{ISM}-(\mu_F+\zeta u_3)F_{ISM}\\
\frac{d_{SH}}{dt} &= (1-\ell)\Lambda_3-\left(\frac{\alpha_I F_{INH}(1-u_1)+\alpha_2 F_{ISM}(1-u_1)+\alpha_3 I_H(1-u_2)+\alpha_4 I_{HT}(1-u_2)+\alpha_5 R_H(1-u_2)}{N_H}\right)S_H-\mu_H S_H\\
\frac{dE_H}{dt} &= \frac{1}{N_H}[\alpha_I F_{INM}(1-u_1)+\alpha_2 F_{ISM}(1-u_1)+\alpha_3 I_H(1-u_2)+\alpha_4 I_{HT}(1-u_2)+\alpha_5 R_H(1-u_2)]-\sigma E_H-\mu_H E_H\\
\frac{dI_H}{dt} &= \ell\Lambda_3+\sigma E_H-\tau_1 I_H-\theta I_H-\partial_1 I_H+\mu_H I_H\\
\frac{dI_{HT}}{dt} &= \theta I_H-\tau_2 I_{HT}-\partial_2 I_{HT}-\mu_H I_{HT}\\
\frac{dR_H}{dt} &= \tau_1 I_H-\tau_2 I_{HT}-\mu_H R_H
\end{aligned}
\right\} \quad (3.1)
$$

We now define our objective function: -

$$
J(u_1,u_2,u_3)=\int_0^T\left(A_1E_H+A_2I_H+A_3N_M+\frac{1}{2}\left(B_1u_1^2+B_2u_2^2+B_3u_3^2\right)\right)dt \qquad (3.2)
$$

Subject to the state system of equations (3.1).

We therefore find the optimal controls that optimize the objective function, similarly we also find the set of controls that minimize the number of Exposed human, Infected human and the total number of Mosquitoes with the associated costs for the implementation of control strategies.

The constants A_1, A_2 and A_3 are the constant associated with the exposed human population, infected human population and the total number of mosquitoes respectively.

The constant, B_1, B_2 and B_3 are the constants of the control variable u_1, u_2 and u_3 respectively.

The terms $\frac{1}{2}B_1u_1^2$, $\frac{1}{2}B_2u_2^2$ and $\frac{1}{2}B_3u_3^2$ are the cost associated with this implementation of the three controls, that is, the use of preventive measures such as insect repellent or mosquito net to reduce contact between human and mosquitoes, abstinence from sexual activities and the use of pesticides at the mosquito breeding sites to eliminate or reduce the number of mosquitoes.

The cost associated with the first control (u_1) refers to the expenses as a result of the use of insect repellent or mosquito net, the cost implication related to the second control (u_2) refers to the expenses associated with the provision of safe sex education and the cost of purchasing condoms. The cost associated with the last control measure (u_3) could be as a result of expenses of using mosquito pesticides and the process of implementation.

Now we assume that u_1^*, u_2^*, u_3^* be the optimal control, we now proceed by finding a set of control functions such that:-

$$J(u_1^*, u_2^*, u_3^*) = \min J(u_1, u_2, u_3), (u_1, u_2, u_3) \in u. \tag{3.3}$$

Subject to the system (3.1).

3.1 Characteristics of control problem

The Pontryagin's maximum principle given in [25] by L.S Pontryagin's in 1956 as a vital result in optimal control was used in this work, the theory that provides a necessary but not sufficient condition that must be satisfied by the optimal solution to optimal control problem, [19]. The Pontryagin maximum principle majorly converts a constrained control problem to unconstrained one by introducing an additional variable to the original problem, [19]. The Pontryagin minimum principle given below, gives the condition under which (x^*, u^*) is optimal.

The following conditions should be met to show or prove the existence of the optimal control problem as presented in [26].

(a) The set of control measures and the corresponding state variables are non-empty.
(b) The set of control is convex and closed
(c) The right hand-hand side of the state system is bounded by a linear function in the state and control variables
(d) The integrand of the objective functional is convex.

Theorem (1): Pontryagin Minimum Principle

Suppose u^* and x^* are optimal for the control problem $\min \int_{t_0}^{t_f} f(x(t), u(t))dt$

Subject to $\frac{dx(t)}{dt} = g(x(t), u(t)), \quad x(t_0) = x_0$

There exist a piecewise differentiable fraction $\lambda(t)$ called the ad joint variable or constant such that the Hamiltonian defined by $H(t, x, u, \lambda) = f(t, x(t), u(t)) + \lambda g(t, x(t), u(t))$ satisfies the following conditions

$$H(t, x, u, \lambda) \leq \quad H(t, x^*, u^*, \lambda) \tag{3.4}$$

$$\frac{dH(t, x, u, \lambda)}{du} = 0 \tag{3.5}$$

$$\frac{d\lambda(t)}{dt} = -\frac{dH(t, x^*, u^*, \lambda)}{dx} \tag{3.6}$$

$$\frac{dx(t)}{dt} = \frac{dH(t, x^*, u^*, \lambda)}{d\lambda} \tag{3.7}$$

For all controls (u) at each time, (t) the ad joint variable (λ) satisfies the transversality condition $\lambda(t_f) = 0$.

Using the Pontryagin minimum principle, we determine first the Lagrangian (L) for the optimal control problem as defined by

$$L = A_1 E_H + A_2 I_H + A_3 N_M + \tfrac{1}{2}\left(B_1 u_1^2 + B_2 u_2^2 + B_3 u_3^2\right) \tag{3.8}$$

And the Hamiltonian (H)

$$\left.\begin{aligned}
&H = L(E_H, I_H, N_M, u_1, u_2, u_3) + \lambda_1 \frac{dA}{dt} + \lambda_2 \frac{dF_M}{dt} + \lambda_3 \frac{dM_M}{dt} + \lambda_4 \frac{dF_{NM}}{dt} + \lambda_5 \frac{dF_{SM}}{dt} + \lambda_6 \frac{dF_{INM}}{dt} + \lambda_7 \frac{dF_{ISM}}{dt} + \\
&\quad \lambda_8 \frac{dS_H}{dt} + \lambda_9 \frac{dE_H}{dt} + \lambda_{10} \frac{dI_H}{dt} + \lambda_{11} \frac{dI_{HT}}{dt} + \lambda_{12} \frac{dR_H}{dt} \\
&= A_1 E_H + A_2 I_H + A_3 N_M + \tfrac{1}{2}\left(B_1 u_1^2 + B_2 u_2^2 + B_3 u_3^2\right) + \lambda_1\{\Lambda_1(F_{NM} + F_{INM})(1-u_3) - \gamma A - (\mu A + \zeta u_3)A - (\mu_\rho + \zeta u_3)A^2\} \\
&+ \lambda_2\{\varphi\gamma A - [\beta_1\rho_\omega + \beta_2\rho_5]F_M - (\partial_M + \zeta u_3)F_M - (\mu_F + \zeta u_3)F_M\} + \lambda_3\{(1-\varphi)\gamma A - (\mu_M + \zeta u_3)M_M\} + \lambda_4\{\beta_1\rho_\omega F_M \\
&- \frac{1}{N_H}(\lambda_1 I_H + \lambda_2 I_{HT})(1-u_1)(\partial_M + \zeta u_3)F_{NM} - (u_F + \zeta u_3)F_{NM}\} + \lambda_5\{\beta_2\rho_5 F_M - \frac{1}{N_H}(\lambda_1 I_H + \lambda_2 I_{HT})(1-u_1)F_{SM} \\
&- (\partial_M + \zeta u_3)F_{SM} - (u_F + \zeta u_3)F_{SM}\} + \lambda_6\{\frac{1}{N_H}(\lambda_1 I_H + \lambda_2 I_{HT})(1-u_1)\mathrm{F_{NM}} - (\partial_M + \zeta u_3)F_{INM} - (u_f + \zeta u_3)F_{INM}\} \\
&+ \lambda_7\{\frac{1}{N_H}(\lambda_1 I_H + \lambda_2 I_{HT})(1-u_1)F_{SM} - (\partial_M + \zeta u_3)F_{ISM} - (u_f + \zeta u_3)F_{ISM}\} + \lambda_8\{(1-\ell)\Lambda_3 \\
&- \frac{1}{\mathrm{N_H}}(\alpha_1 F_{INM}(1-u_1) + \alpha_2 F_{ISM}(1-u_1) + \alpha_3 I_H(1-u_2) + \alpha_4 I_{HT}(1-u_2) + \alpha_s R_H(1-u_2))\mathrm{S_H} - \mu_H \mathrm{S_H}\} \\
&+ \lambda_9\{\frac{1}{\mathrm{N_H}}[\alpha_1 F_{INM}(1-u_1) + \alpha_2 F_{ISM}(1-u_1) + \alpha_3 I_H(1-u_2) + \alpha_4 I_{HT}(1-u_2) + \alpha_s R_H(1-u_2)]\mathrm{S_H} - \sigma E_H - \mu_H E_H \\
&+ \lambda_{10}\{\ell\Lambda_3 + \sigma E_H - \tau_1 I_H - \theta I_H - \partial_1 I_H + \mu_H I_H\} + \lambda_{11}\{\theta I_H - \tau_2 I_{HT} - \partial_2 I_{HT} - \mu_H I_{HT}\} + \lambda_{12}\{\tau_1 I_H - \tau_2 I_{HT} - \mu_H R_H\}
\end{aligned}\right\} \tag{3.9}$$

Where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11}$ & λ_{12} are the ad joint variables satisfying the following ad joint system:

$$
\left.
\begin{aligned}
\lambda_1^1 &= -\frac{\partial H}{\partial A} = \gamma + \lambda_1[(\mu_A + \zeta u_3) - 2(\mu_\rho + \zeta u_3)A] - \lambda_2\phi\gamma - \lambda_3\gamma(1-\phi) \\
\lambda_2^1 &= -\frac{\partial H}{\partial F_M} = \lambda_2\{-\phi\gamma A(1-u_3) + [\beta_1\rho_\omega + \beta_2\rho_S] + (\partial_M + \zeta u_3) + (\mu_F + \zeta u_3)\} + \lambda_4\beta_1\rho_\omega - \lambda_5\beta_2\rho_S \\
\lambda_3^1 &= -\frac{\partial H}{\partial M_M} = \lambda_3(\mu_M + \zeta u_3) \\
\lambda_4^1 &= -\frac{\partial H}{\partial F_{NM}} = -\lambda_1\Lambda_1(1-u_3) + \frac{\lambda_4}{N_H}\{(\lambda_1 I_H + \lambda_2 I_{HT})(1-u_1) + (\partial_M + \zeta u_3) + (\mu_F + \zeta u_3)\} - \frac{\lambda_6}{N_H}\{(\lambda_1 I_H + \lambda_2 I_{HT})(1-u_1)\} \\
\lambda_5^1 &= -\frac{\partial H}{\partial F_{SM}} = \lambda_5\{\frac{1}{N_H}(\lambda_1 I_H + \lambda_2 I_{HT})(1-u_1) + (\partial_m + \zeta u_3) + (\mu_F + \zeta u_3)\} - \lambda_7\{\frac{1}{N_H}(\lambda_1 I_H + \lambda_2 I_{HT})(1-u_1)\} \\
\lambda_6^1 &= -\frac{\partial H}{\partial F_{INM}} = -\lambda_1\Lambda_1(1-u_3) + \lambda_6[(\partial_M + \zeta u_3) + (\mu_F + \zeta u_3)] + \frac{\alpha_1\lambda_8 S_H}{N_H}(1-u_1) - \frac{\alpha_1\lambda_9 S_H}{N_H}(1-u_1) \\
\lambda_7^1 &= -\frac{\partial H}{\partial F_{ISM}} = \lambda_7[(\partial_M + \zeta u_3) + (\mu_F + \zeta u_3)] + \frac{\lambda_8\alpha_2}{N_H}(1-u_1)S_H - \frac{\lambda_9\alpha_2}{N_H}(1-u_1)S_H \\
\lambda_8^1 &= -\frac{\partial H}{\partial S_H} = \lambda_8\{\frac{1}{N_H}[\alpha_1 F_{INM}(1-u_1) + \alpha_2 F_{ISM}(1-u_1) + \alpha_3 I_H(1-u_2) + \alpha_4 I_{HT}(1-u_2) + \alpha_5 R_H(1-u_2)] - \mu_H\} \\
&\quad - \{\frac{1}{N_H}[\alpha_1 F_{INM}(1-u_1) + \alpha_2 F_{ISM}(1-u_1) + \alpha_3 I_H(1-u_2) + \alpha_4 I_{HT}(1-u_2) + \alpha_5\lambda_9 R_H(1-u_2)]\} \\
\lambda_9^1 &= -\frac{\partial H}{\partial E_H} = -A_1 + \sigma + \mu_H - \lambda_{10}\sigma \\
\lambda_{10}^1 &= -\frac{\partial H}{\partial I_H} = -A_2 + \frac{\lambda_4\lambda_1}{N_H}(1-u_1)F_{NM} + \frac{\lambda_5}{N_H}\lambda_1(1-u_1)F_{SM} - \frac{\lambda_6}{N_H}\lambda_1(1-u_1)F_{NM} - \lambda_7\frac{\lambda_1}{N_H}(1-u_1)F_{SM} - \frac{\lambda_8\alpha_3}{N_H}(1-u_2)S_H \\
&\quad - \frac{\lambda_9\alpha_3}{N_H}(1-u_2)S_H - \lambda_{10}(\mu_H + \partial_1 + \theta + \tau_1) - \lambda_{11}\theta - \lambda_{12}\tau_1 \\
\lambda_{11}^1 &= -\frac{\partial H}{\partial I_{HT}} = \frac{\lambda_4\lambda_2}{N_H}[(1-u_1)F_{NM}] + \frac{\lambda_5}{N_H}\lambda_2(1-u_1)F_{SM} - \frac{\lambda_6\lambda_2}{N_H}(1-u_1)F_{NM} - \frac{\lambda_2\lambda_7}{N_H}(1-u_1)F_{SM} + \frac{\lambda_8\alpha_3}{N_H}(1-u_2)S_H \\
&\quad - \frac{\lambda_9\alpha_4}{N_H}(1-u_2)S_H + \lambda_{11}(\tau_2 + \partial_2 + \mu_H) + \lambda_2\tau_2 \\
\lambda_{12}^1 &= -\frac{\partial H}{\partial R_H} = \frac{\lambda_8\alpha_5}{N_H}(1-u_2)S_H + \lambda_9\alpha_5(1-u_2)S_H + \lambda_{12}\mu_H
\end{aligned}
\right\} \quad (3.91)
$$

The transversality condition or boundary conditions are $\lambda_i(T) = 0, i = 1, 2, \ldots 12$ Where T is the end of time Period.

By the optimality conditions, we have

$$\frac{\partial H}{\partial u_i} = 0, \quad i = 1,2,3 \text{ at } u_i \Rightarrow u_i^*$$

thus,

$$\left.\begin{aligned}
&\frac{\partial H}{\partial u_1} = B_1 u_1 + \frac{\lambda_4}{N_H}\left[(\lambda_1 I_H + \lambda_2 I_{HT})\right]F_{NM} + \frac{\lambda_5}{N_H}\left[(\lambda_1 I_H + \lambda_2 I_{HT})\right]F_{SM} - \frac{\lambda_6}{N_H}\left[(\lambda_1 I_H + \lambda_2 I_{HT})\right]F_{NM} - \\
&\frac{\lambda_7}{N_H}\left[(\lambda_1 I_H + \lambda_2 I_{HT})\right]F_{SM} + \frac{\lambda_8}{N_H}\left[(\alpha_1 F_{INM} - \alpha_2 F_{ISM})\right]S_H - \frac{\lambda_9}{N_H}\left[(\alpha_1 F_{INM} + \alpha_2 F_{ISM})\right]S_H = 0 \\
&\frac{\partial H}{\partial u_2} = B_2 u_2 + \lambda_8\left[\frac{1}{N_H}(\alpha_3 I_H + \alpha_4 I_{HT}) + \alpha_5 R_H\right]S_H - \frac{\lambda_9}{N_H}\left[\alpha_3 I_H + \alpha_4 I_{HT} + \alpha_5 R_H\right]S_H = 0 \\
&\frac{\partial H}{\partial u_3} = B_3 u_3 - \lambda_1\left[\Lambda_1(F_{NM} + F_{INM}) + A\zeta + A\zeta^2\right] - 2\lambda_2\zeta F_M - 2\lambda_3\zeta M_M - 2\lambda_4\zeta F_{NM} - 2\lambda_5\zeta F_{SM} - 2\lambda_6\zeta F_{INM} - 2\lambda_7\zeta F_{ISM} = 0
\end{aligned}\right\} \quad (3.92)$$

We obtain now the solution by making u_1, u_2, u_3 subject of the formula from system (3.92).

$$\left.\begin{aligned}
u_1^* &= \frac{1}{B_1 N_H}[(\lambda_9 - \lambda_8)(\alpha_1 F_{INM} + \alpha_2 F_{ISM})S_H + (\lambda_1 I_H + \lambda_2 I_{HT})(\lambda_7 F_{NM} + \lambda_6 F_{NM} - \lambda_5 F_{SM} - \lambda_4 F_{NM})] \\
u_2^* &= \frac{1}{B_2 N_H}[(\alpha_3 I_H + \alpha_4 I_{HT} + \alpha_5 R_H)(\lambda_9 S_H - \lambda_8 S_H)] \\
u_3^* &= \frac{1}{B_3}[\lambda_1(\Lambda_1(F_{NM} + F_{INM}) - \gamma A + 3A\zeta) + 2\zeta(\lambda_2 F_M + \lambda_3 M_M) + \frac{2\zeta}{N_H}(\lambda_4 F_{NM} + \lambda_5 F_{SM} + \lambda_6 F_{INM} + \lambda_7 F_{ISM})]
\end{aligned}\right\} \quad (3.93)$$

Taking the second partial derivatives of equation (3.92) we obtained:

$\frac{\partial^2 H}{\partial u_1^2} = B_1 > 0$, $\frac{\partial^2 H}{\partial u_2^2} = B_2 > 0$, $\frac{\partial^2 H}{\partial u_3^2} = B_3 > 0$, Since the second partial derivatives of (H) with respect to u_1, u_2, u_3 are greater than zero(0), then the control is associated with minimizing a problem which has to do with the reduction of the number of Exposed human, Infected human and the total number of mosquitoes with the associated cost for the implementation of control strategies.

4 Discussion and Model Simulations

This research is aimed at studying the Impact of Optimal Control measures on the Dynamics of zika Virus Model using the Sterile Insect Technology. The optimal control measures (u_1, u_2, u_3) are parameter reliant, and as such, for us to check the magnitude of these control measures, we solved the system numerically using the maple software. The parameter values used are tabulated in Table 3. The weight constants are given as A_1=0.07, A_2=0.07, A_3=0.07, B_1=20, B_2=20, B_3=50. We assume that the cost of implementation related with the first control u_1, is the same as that related with the second control u_2. B_3 is more than B_1 and B_2 ,this is subject to the fact that vector reduction or elimination is more costly to implement than the implementation of the use of insect repellent or mosquito net (u_1) and the expenses

linked with the provision of sex education through enlightenment campaign program which is represented by (u_2).

The use of Insect repellant or mosquito net is less expensive and easy to implement, in the same way, the provision of safe sex education is easier and less expensive to implement.

Implementing the three (3) control strategies by simulating the three different cases to the optimality system, with control measures and without control measures, it is therefore evident from Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13, that the implementation of optimal control measures gives good results in the sense that it reduces the infected and infectious population of both the mosquito and that of human classes. Model variables are assumed to be;

[A , M_M , F_M , F_{NM} , F_{SM} , F_{INM} , F_{ISM} , S_H, E_H , I_H , I_{HT} , R_H]=[2500,160,500,250,120,125,40,1000,30,20,15,0].The parameter values used for the model simulation are tabulated in Table 3 below:

Table 3. Numerical values of parameters used

S/N	Parameters	Parameter values	Source
1	Λ_1	120	[14]
2	ϕ	0.6	[14]
3	β_1	0.5	Assumed
4	β_2	0.6	Assumed
5	γ	0.06	Assumed
6	μ_M	0.26	Assumed
7	μ_ρ	0.00002	[10]
8	μ_H	0.00005	[11]
9	μ_A	0.25	[11]
10	μ_F	0.27	Assumed
11	∂_M	0.03	Assumed
12	ρ_W	0.6	Assumed
13	ρ_S	0.4	Assumed
14	∂_1	0.002	Assumed
15	∂_2	0.001	Assumed
16	τ_1	0.14	[2]
17	θ	0.002	Assumed
18	τ_2	0.016	Assumed
19	ℓ	0.05	Assumed
20	σ	0.03	[8]
21	Λ_3	40	Assumed
22	α_1	0.0002	Assumed
23	α_2	0.0001	Assumed
24	α_3	0.09	[14]
25	α_4	0.07	Assumed

26	α_5	0.05	[6]
27	λ_1	0.09	Assumed
28	λ_2	0.07	Assumed

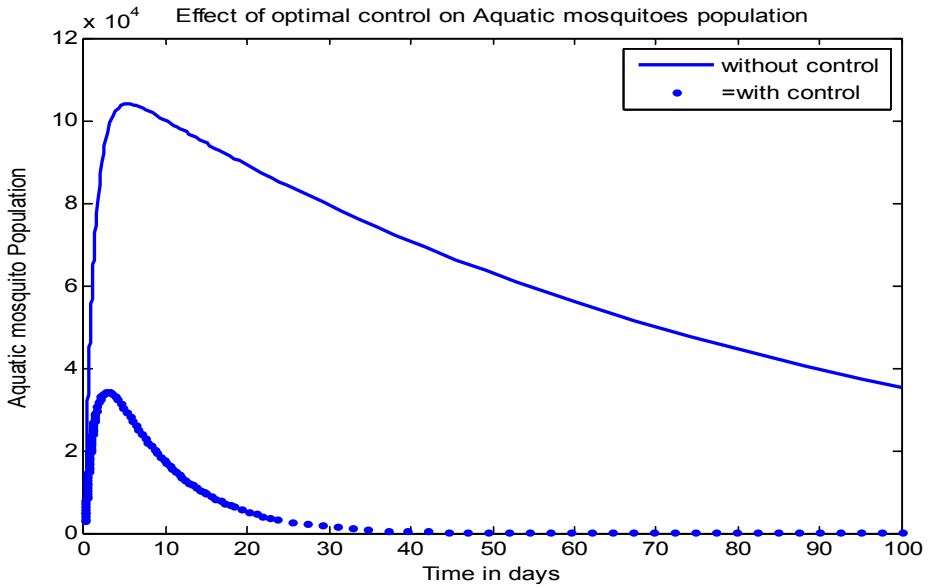

Fig. 2. Effect of optimal control on A class

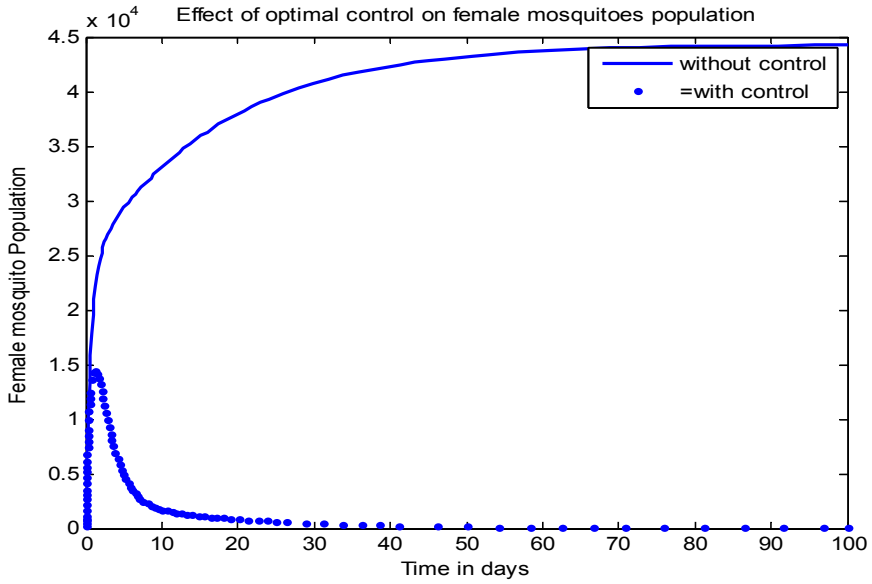

Fig. 3. Effect of optimal control on F_M class

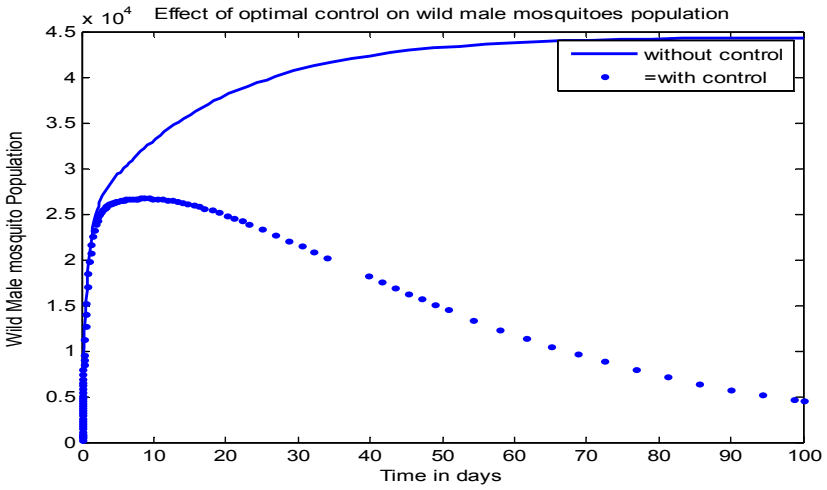

Fig. 4. Effect of optimal control on M_M class

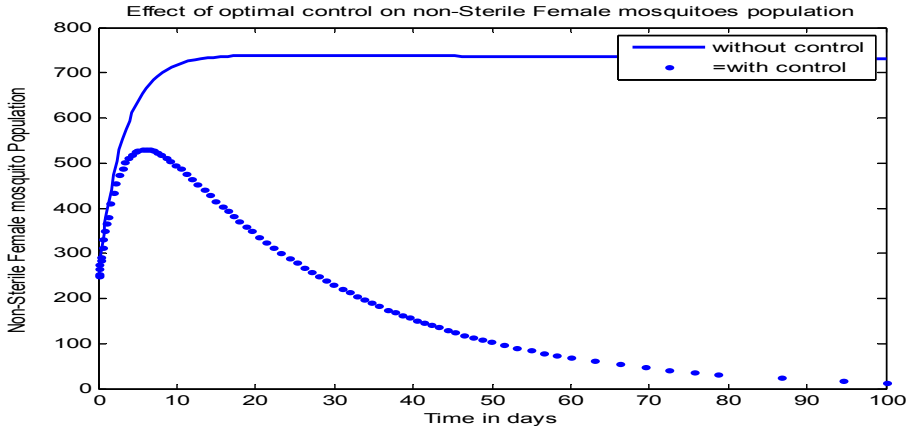

Fig. 5. Effect of optimal control on the F_{NM} class

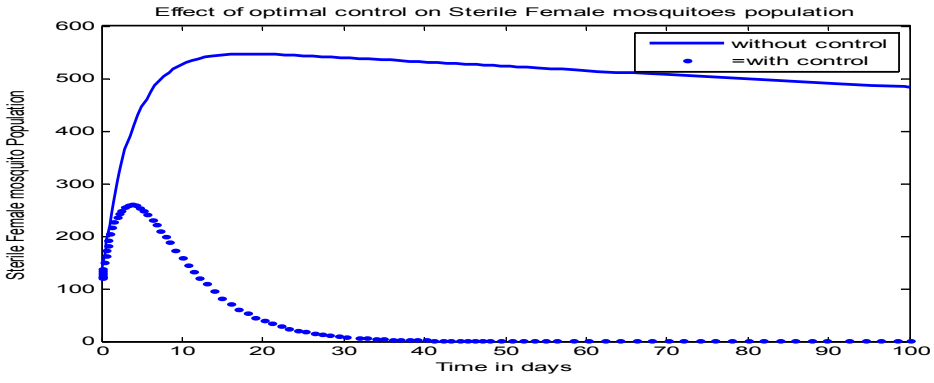

Fig. 6. Effect of optimal control on the F_{SM} class

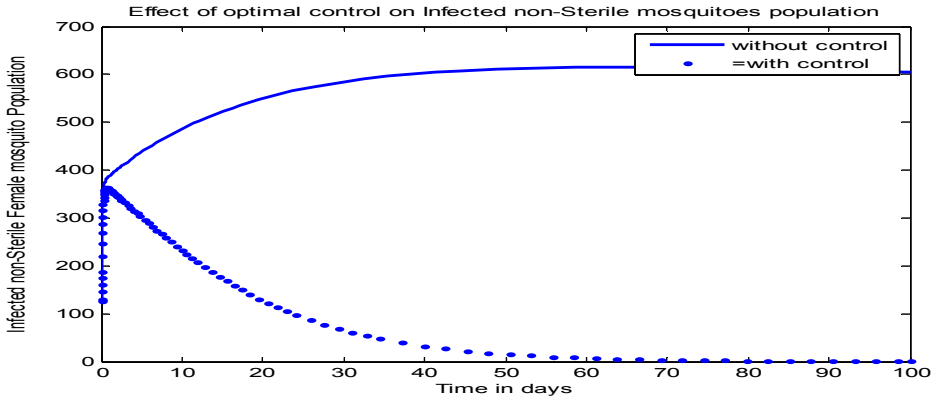

Fig. 7. Effect of optimal control on the F_{INM} class

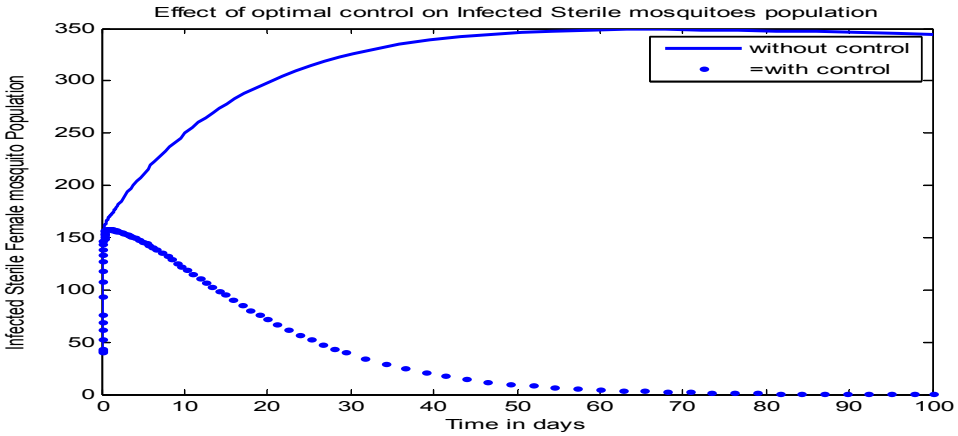

Fig. 8. Effect of optimal control on the F_{ISM} class

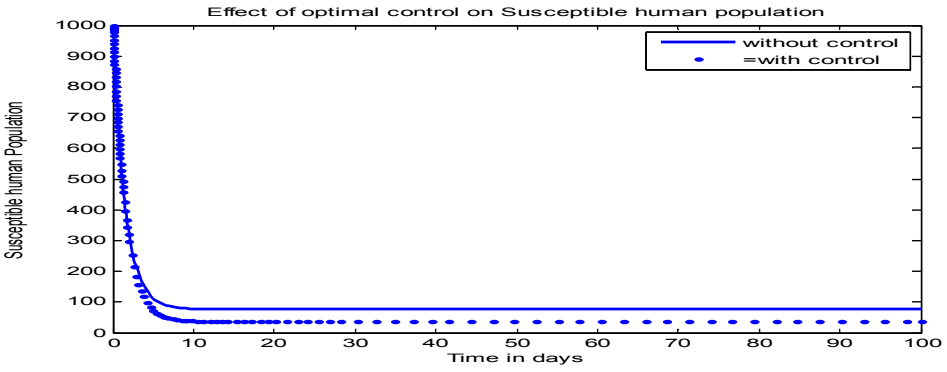

Fig. 9. Effect of optimal control on Susceptible human

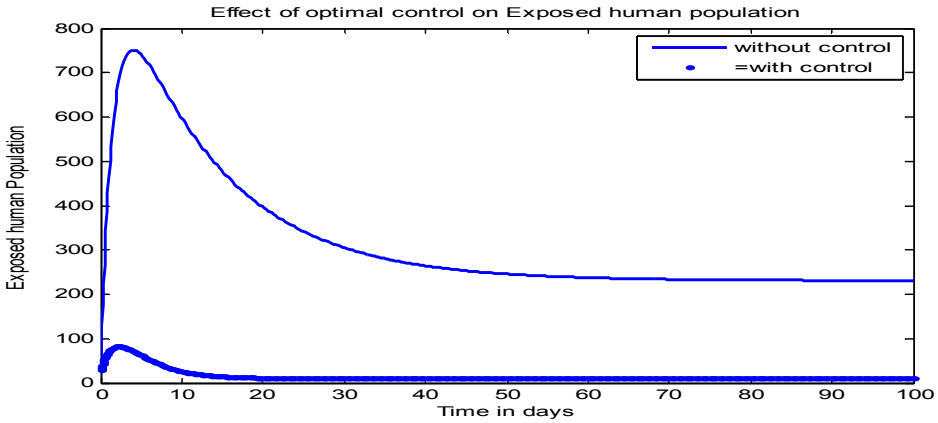

Fig. 10. Effect of optimal control on the exposed human

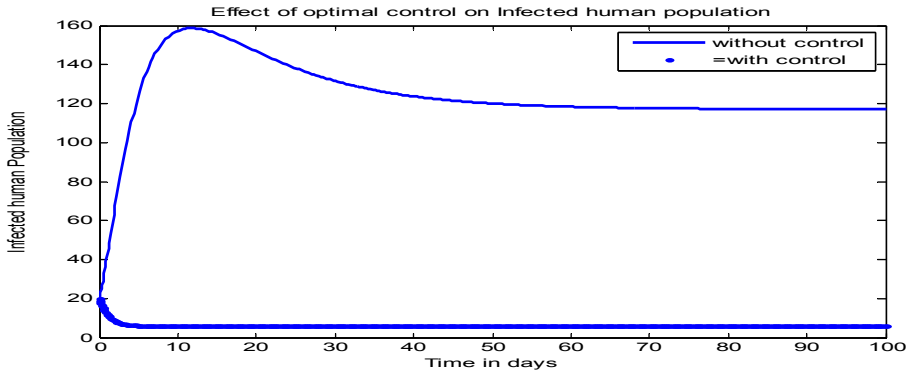

Fig. 11. Effect of optimal control on the infected human

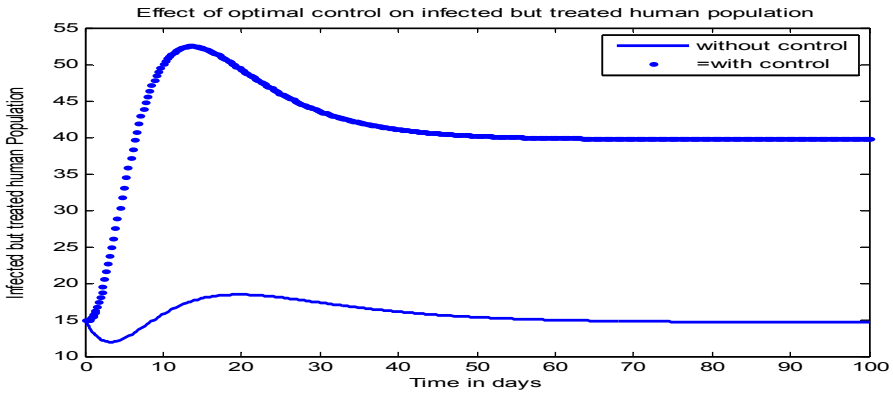

Fig. 12. Effect of optimal control on Infected but treated

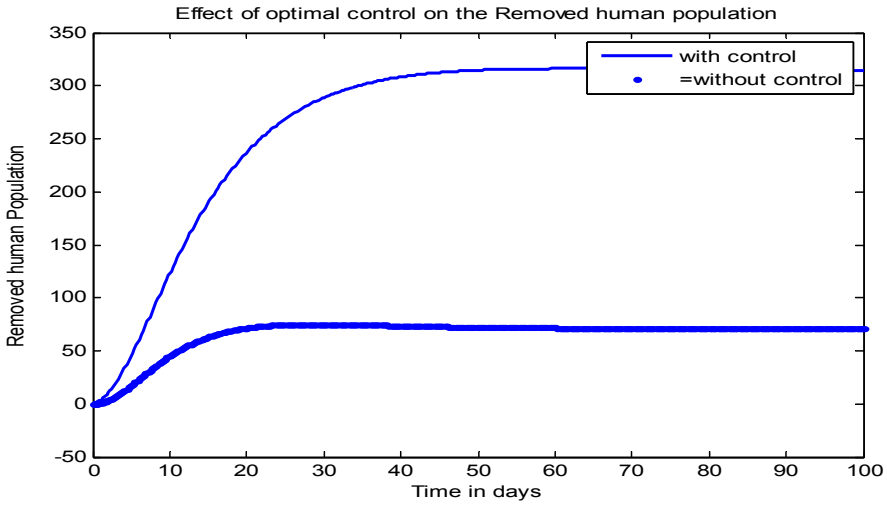

Fig. 13. Effect of optimal control on the Removed human

5 Conclusion

In this article, we formulated a mathematical model for the transmission dynamics of zika virus infection using the Sterile Insect Technology. The model is now extended by incorporating the impact of Optimal Control Strategies on the transmission dynamics. We proposed three (3) control measures that reduce the number of Exposed human, infected human and the total number of Mosquitoes. This control strategy is aimed at reducing the zika virus infection by reducing contacts between vector-to-human, human to human and vector elimination. The Pontryagin's maximum principle was then used to find the necessary conditions, to also determine the optimality system of our model and to finally obtain solution to the control problem. We then performed numerical simulations to compare the results of the system with optimal control measures and that without control measures. Result shows that the optimal control measure is more effective to reduce the number of the Exposed human, the infected human population and above all, to reduce the total number of mosquito population that cause zika virus disease. This result implies that zika virus infection transmission from vector to human and human-to-human can be reduced drastically.

Competing Interests

Authors have declared that no competing interests exist.

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