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Unsaturated Stream-Aquifer Connection When the Riverbed Itself Desaturates

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

In a previous series of 3 articles in this Journal we have looked at the situation when the aquifer desaturates while the riverbed (clogging layer) remained saturated. However it is quite possible that the riverbed itself will desaturate, particularly if the river stage is low and especially when the river goes dry. In this article we look at the situation when indeed the riverbed desaturates as the capillary pressure at the interface between the bottom of the riverbed and the top of the unsaturated zone in the aquifer exceeds the riverbed entry pressure, while the river itself does not go dry. Utilizing different soil textures for a riverbed we found that the seepage rate, following a slight increase in value, tends rapidly to an asymptotic limit which is not much higher than the seepage value at incipient desaturation of the riverbed.

Thus the assumption that the riverbed does not desaturate may lead to grossly exaggerated values of the seepage rate.

Keywords: Stream-aquifer interaction; flow exchange; stream depletion; unsaturated hydraulic connection; analytical coupling.

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1. INTRODUCTION

In a previous article [1] we have discussed at length the motivation for this ongoing research with a review of the historical motivation in the United States for the development of most groundwater (or integrated hydrologic) simulation models in wide use today for large-scale regional studies. In several articles the limitations of these codes were described in great details [2,3,4]. In this article we inquire about the impact of desaturation within the riverbed itself and how it affects the overall seepage and recharge rates to the unsaturated zone in the aquifer and then the recharge rate to the aquifer itself.

2. THE CASE OF UNSATURATED CONNECTION IN THE AQUIFER

In this figure (and in the text) H is the river stage, B is the half-width of the river bottom and G is the grid size in the lateral direction.

Once an unsaturated connection is established an additional component is added to the system: the unsaturated zone above the capillary fringe.

The case of desaturation within the aquifer has been discussed in great details in previous articles [3,4]. What needs to be emphasized is that in this situation the capillary drive is a force that resists the intrusion of seepage water as the water content profile in Fig. 2 shows clearly.

In the absence of gravity water flows from low toward high capillary pressure, from high water content to low water content, in other words is attracted by suction. In the unsaturated zone what allows seepage downward is the force of gravity overpowering that of capillarity. Fig. 2 illustrates the water content profile shape under the river and the notations used in the unsaturated zone and the water table mound.

The word *interface* refers to the boundary between the bottom of the clogging layer and the top of the underlying aquifer. We use the term *capillary zone* for the combination of both the *unsaturated zone* and the *capillary fringe*.

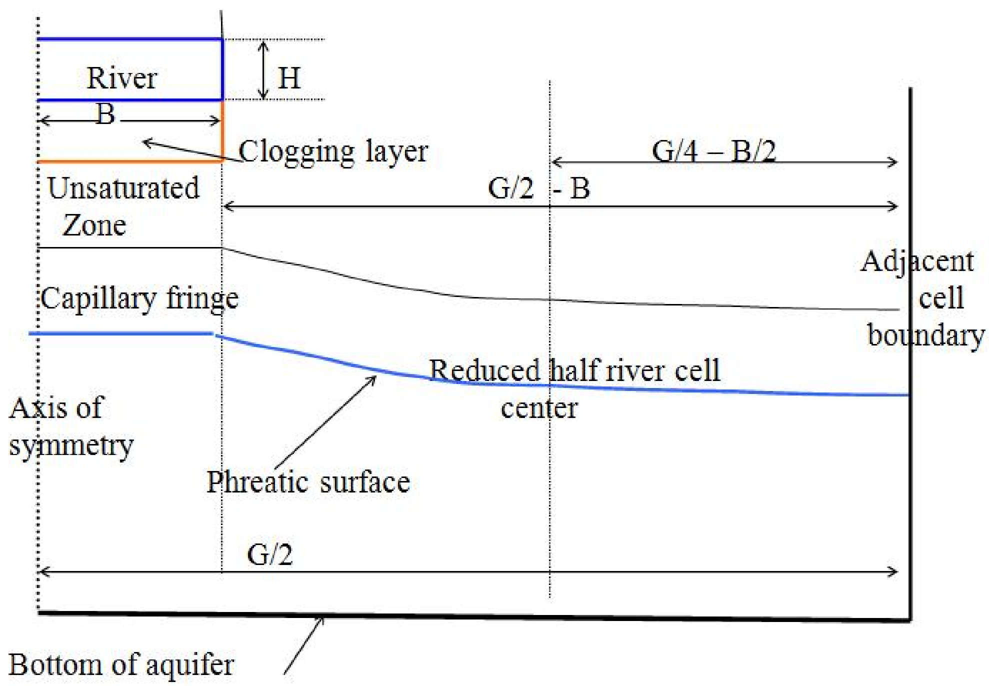

Fig. 1. Schematic view of the different components of the system

Under an unsaturated connection if we knew the capillary pressure at the interface, h_{cI}, then we would know the seepage velocity at the interface, i_S, given by Darcy's law:

$$i_S = K_{cl} \frac{(H + h_{cI} + e_{cl})}{e_{cl}} \quad (1)$$

K_{cl} is the conductivity of the clogging layer and e_{cl} is its thickness. The flow from the clogging layer is given by Eq.(1) under the assumption that the capillary pressure at the interface does not exceed its entry pressure. However, if it does, Eq.(1) is no longer applicable because the riverbed itself desaturates.

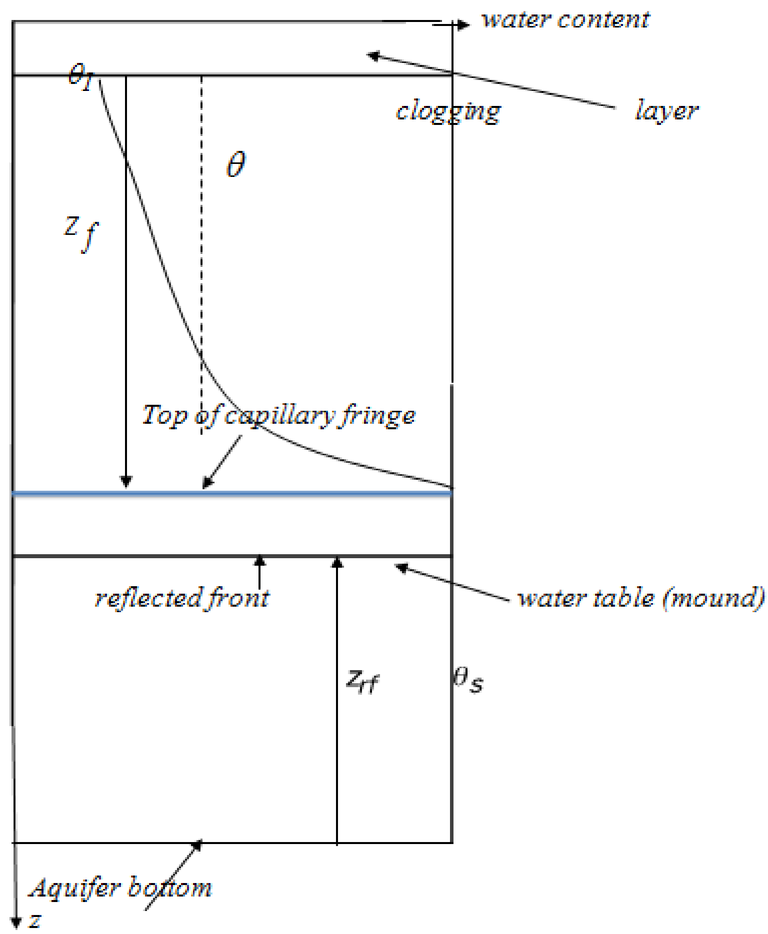

Fig. 2. Water content profile below the river

In this figure (and in the text) θ_I is the water content at the interface between the bottom of the clogging layer and the top of the aquifer below, θ is the mean water content in the unsaturated zone, z_f is the depth (thicknes) of the unsaturated zone, θ_S is the saturated water content and z_{rf} is the height of the phreatic surface above the bottom of the aquifer

3. THE CASE OF UNSATURATED CONNECTION WITHIN THE RIVERBED ITSELF

In this article we consider only the case that there is water in the river with a stage H, that is the maximum depth of water in the river cross-section. In other words the river is not going dry. (All parameters refer to the clogging layer and for brevity the suffix cl is not added to the names of the variables)

A reasonable assumption is that the entry pressure in the clogging layer exceeds that of the aquifer material beneath. As long as the interface capillary pressure is less than the entry pressure, h_{ce}, of the clogging layer, then the riverbed does not desaturate.

Thus we are only concerned with the case when $h_{cI} \geq h_{ce}$.

A practical assumption is that the riverbed thickness exceeds its entry pressure $e \geq h_{ce}$.

Under steady-state conditions then the seepage rate is the velocity throughout the riverbed and it is uniform through the profile. Darcy's law gives:

$$i_S = Kk_{rw}[1+\frac{dh_c}{dz}] \quad \text{or} \quad \frac{i_S}{K} = i_S^* = k_{rw}[1+\frac{dh_c}{dz}] \tag{2}$$

K is the conductivity of the aquifer, k_{rw} is the relative conductivity, h_c is the capillary pressure expressed as an equivalent height of water and z is the vertical coordinate oriented positive downward.

$$\text{In the zone where } h_c \leq h_{ce} \text{ since } k_{rw} = 1 \text{ then } \frac{dh_c}{dz} = i_S^* - 1 \tag{3}$$

Integration over the domain of capillary fringe provides the thickness of the capillary fringe:

$$z_e = \frac{h_{ce} + H}{i_S^* - 1} \tag{4}$$

Clearly the seepage rate exceeds the conductivity of the riverbed.

Under static conditions the thickness of the capillary fringe is the entry pressure. It is not the case under a flowing condition. Given that the flow in the capillary fringe is saturated, if the downward flow exceeds what it would be by gravity alone the gradient of head cannot be unity $\frac{h_{ce}}{h_{ce}}$ but must exceed it and be $\frac{h_{ce}}{z_e}$ wth $z_e \leq h_{ce}$ in case of downward flow. In case of upward flow then $z_e \geq h_{ce}$.

Beyond that zone, defining the capillary drive as:

$$H_c(h_c) = \int_0^{h_{ce}} k_{rw} dh_c + \int_{h_{ce}}^{h_c} k_{rw} dh_c = h_{ce} + \int_{h_{ce}}^{h_c} e^{-(\frac{h_c - h_{ce}}{H_{cS}})} dh_c \tag{5}$$

More explicitly:

$$H_c(h_c) = h_{ce} + H_{cS}[1 - e^{-(\frac{h_c - h_{ce}}{H_{cS}})}] = h_{ce} + H_{cS}[1 - k_{rw}] \tag{6}$$

By definition and from above

$$\frac{dH_c(h_c)}{dz} = k_{rw}\frac{dh_c}{dz} = -H_{cS}\frac{dk_{rw}}{dz} \tag{7}$$

The governing equation is thus: $$i_S^* = k_{rw} - H_{cS}\frac{dk_{rw}}{dz} \tag{8}$$

More explicitly: $$\frac{d(z - z_e)}{H_{cS}} = \frac{dk_{rw}}{k_{rw} - i_S^*} \tag{9}$$

Specifically after integration from the bottom of the capillary fringe

$$z - z_e = H_{cS}\ln[\frac{i_S^* - k_{rw}(h_c)}{i_S^* - 1}] \tag{10}$$

Solving for k_{rw} yields: $$k_{rw}(h_c) = i_S^* - (i_S^* - 1)e^{\frac{z - \frac{h_{ce}}{i_S^* - 1}}{H_{cS}}} \tag{11}$$

One can verify that at $z = z_e$ and $h_c = h_{ce}$, k_{rw} takes the value 1 as it should.
Integration to the bottom of the riverbed yields:

$$e - z_e =' H_{cS}\ln[\frac{i_S^* - k_{rw}(h_{cI})}{i_S^* - 1}] \tag{12}$$

More explicitly: $$e - \frac{h_{ce} + H}{i_S^* - 1} =' H_{cS}\ln[\frac{i_S^* - k_{rw}(h_{cI})}{i_S^* - 1}] \tag{13}$$

One can verify that for $h_{cI} = h_{ce}$ then $i_S = K[\frac{H + h_{ce} + e)}{e}$ as it should.

For a given value of h_{cI} then $k_{rw}(h_{cI})$ is known and that nonlinear equation Eq.(13) provides the solution for the seepage rate.

Once that seepage rate is known one can calculate the mean relative permeability in the unsaturated zone within the riverbed and deduce from it the mean water content in that zone. In the capillary fringe it is water content at saturation, θ_{sat}. In the unsaturated zone the relative permeability is:

$$\bar{k}_{rw} = \frac{1}{e - z_e} \int_{z_e}^{e} \{ i_S^* - (i_S^* - 1) e^{\frac{z - z_e}{H_{cS}}} \} dz \tag{14}$$

$$\bar{k}_{rw} = i_S^* - \frac{(i_S^* - 1) H_{cS}}{e - z_e} [e^{\frac{z - z_e}{H_{cS}}}] \Bigg|_{z_e}^{e} = i_S^* - \frac{(i_S^* - 1) H_{cS}}{e - z_e} [e^{\frac{e - z_e}{H_{cS}}} - 1] \tag{15}$$

Once that value obtained the mean normalized water content in the unsaturated zone is:

$$\bar{\theta}^* = (\bar{k}_{rw})^{\frac{1}{p}} \quad \text{and} \tag{16a}$$

$$\bar{\theta} = (\theta_{sat} - \theta_{res}) \bar{\theta}^* + \theta_{res} \tag{16b}$$

The overall average water content in the river bed is:

$$\bar{\theta}_e = \frac{z_e \theta_{sat} + (e - z_e) \bar{\theta}}{e} \tag{17}$$

During a period of time, Δt, as the interface capillary pressure continues to increase the riverbed will have drained and the drainage rate is:

$$\Delta V_D = \frac{e(\bar{\theta}_e^o - \bar{\theta}_e)}{\Delta t} \tag{18}$$

$\bar{\theta}_e^o$ is the value at the beginning of the period, the old value. The interface velocity is:

$$v_I = i_S + \frac{e(\bar{\theta}_e^o - \bar{\theta}_e)}{\Delta t} \tag{19}$$

When the clogging layer is fully saturated then the velocity at the interface is the seepage rate but it is no longer the case when the clogging layer desaturates. However the difference is likely to be small because the thickness of the riverbed is small compared to that of the unsaturated zone in the aquifer below the riverbed.

From a numerical solution point of view the most laborious task for implementing this scheme is the solution of the nonlinear Eq.(13) for the normalized seepage rate, namely

$$e - \frac{h_{ce} + H}{i_S^* - 1} =' H_{cS} \ln[\frac{i_S^* - k_{rw}(h_{cI})}{i_S^* - 1}] \tag{13}$$

As the value of h_{cI} increases from the original value of the entry pressure, h_{ce}, the right hand-side will increase from the value of zero and to match that increase on the right hand-side the value of z_e

on the left hand-side must decrease and the seepage rate must increase. This is a rather delicate iterative procedure.

Runs were made for 11 different soil types. Because the patterns displayed for the 11 soil textures were the same, showing a rather quick leveling of the seepage rate as the interface capillary pressure increases, only three figures are shown, two for the extreme soil textures and one for the middle range. In that sense Figs. 3, 4 and 5 thus illustrate the results for all soil textures.

Table 1. Hydrologic soil properties classified by soil texture

Symbols: h_{ce} is drainage entry pressure in cm, λ is Brooks-Corey pore size distribution, $M = \dfrac{1}{\lambda}$, $p = 3 + 2M$, $H_{cS} = h_{ce}M/(p - M)$ in cm, K is saturated hydraulic conductivity in m/day, θ_{sat} is saturated water content, θ_{res} is residual water content.

Soil type	h_{ce}	λ	M	p	H_{cS}	K (m/day)	θ_{sat}	θ_{res}
Sand	15.98	0.69	1.44	5.88	5.185	5.0400	0.4370	0.0200
Loamy sand	20.58	0.55	1.81	6.62	7.740	1.4664	0.4370	0.0350
Sandy loam	30.20	0.38	2.65	8.29	14.152	0.6216	0.4430	0.0410
Loam	40.12	0.25	3.97	10.94	22.847	0.1632	0.4630	0.0270
Silty loam	50.87	0.23	4.27	11.55	29.888	0.3168	0.5010	0.0150
Sand clay loam	59.61	0.32	3.13	9.27	30.460	0.1032	0.3980	0.0680
Clay loam	56.43	0.24	4.13	11.26	32.694	0.0552	0.4640	0.0750
Silty clay loam	70.33	0.18	5.65	14.30	45.937	0.0360	0.4710	0.0400
Sandy clay	79.48	0.22	4.48	11.97	47.621	0.0288	0.4300	0.1090
Silty clay	76.54	0.15	6.67	16.33	52.786	0.0216	0.4790	0.0560
Clay	85.60	0.17	6.06	15.12	57.258	0.0144	0.4750	0.0900

Source: Morel-Seytoux, 1989 [5]

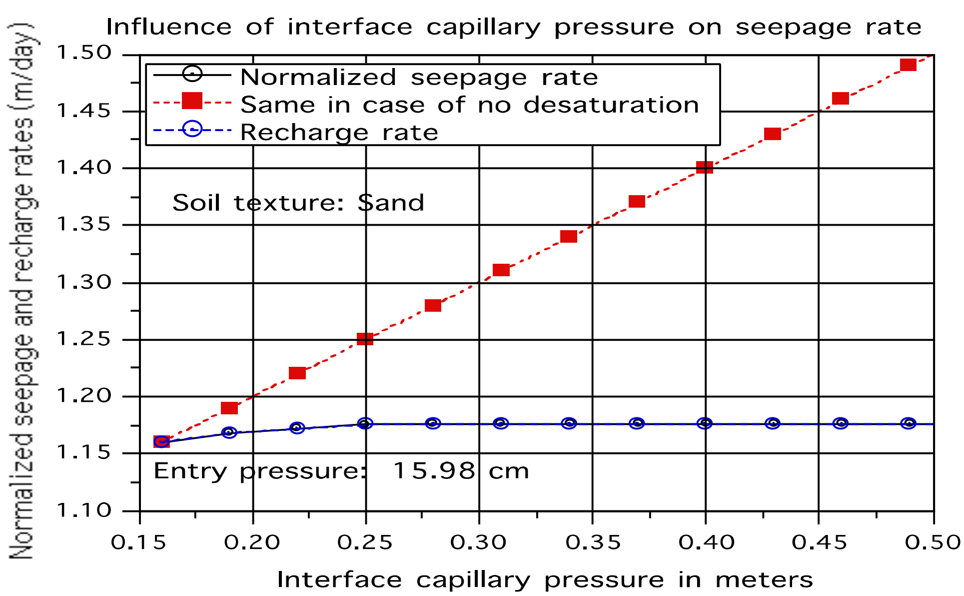

Fig. 3. Influence of desaturation on the seepage rate for a sand

The seepage rate when the riverbed desaturates never exceeds the value when the riverbed remains saturated. Eventually as the interface capillary pressure keeps increasing the seepage rate tends to level off. In practice what causes such increase in capillary pressure is a drop in the water-table. In other words after a while further drop in water-table does not lead to an increase in seepage. This is very clear for the sand in Fig. 1, quite clear for the sand clay loam and fairly clear for the clay.

The entry pressure is not likely to ever exceed 1.0 meter and probably much less. The velocity at the interface is essentially the same as the seepage rate. This is due in part because the assumed riverbed thickness is only 1.0 m. But even for a greater thickness it is unlikely to add much to the seepage rate. Very clearly the assumption that the clogging layer remains always saturated is not realistic and would lead to exaggerated values for the seepage rate.

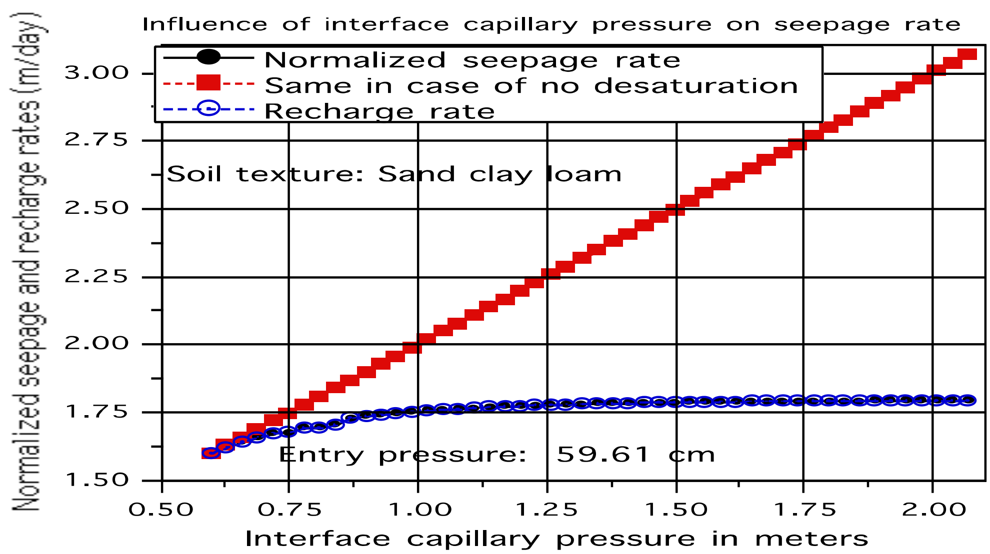

Fig. 4. Influence of desaturation on the seepage rate for a sand clay loam

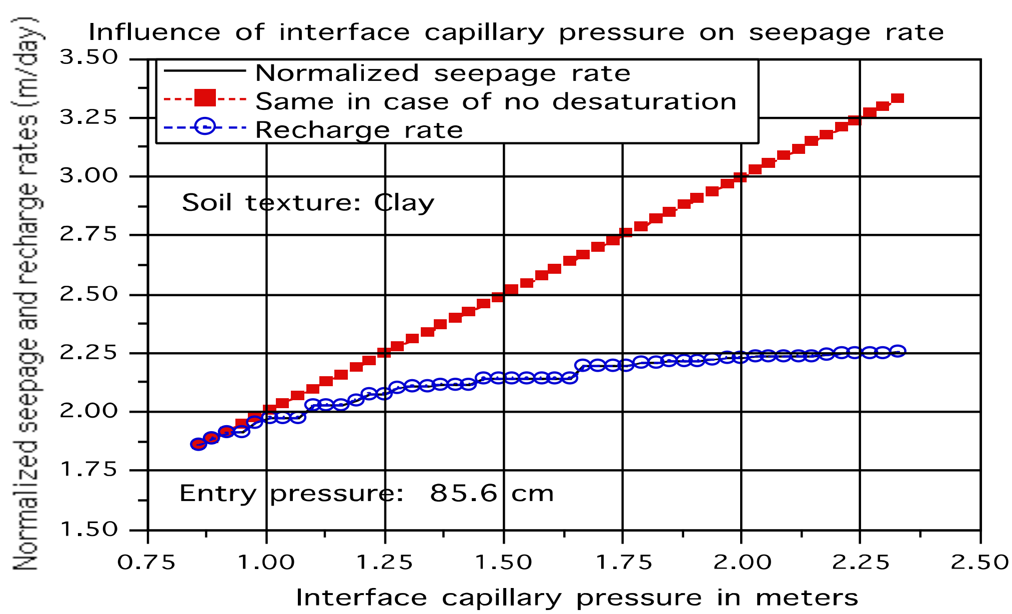

Fig. 5. Influence of desaturation on the seepage rate for a clay

In the case of the sand the limiting value of seepage rate was only about 2% higher than the seepage rate at incipient desaturation. For the sand clay loam it was about 10% more and for the clay it was about 16% more. As a rough rule of thumb the percentage increases in almost direct proportion to the entry pressure.

4. DISCUSSION

The riverbed will desaturate if the capillary pressure at the interface exceeds its entry pressure. We have seen that as the capillary pressure increases the seepage rate fairly quickly stops to increase, and everything else remaining the same, tends to a limiting constant value. Once that seepage rate and the interface velocity known, then the procedures for the determination of the evolution of the variables in the unsaturated zone of the aquifer are the same as discussed in the previous articles [3,4]. However it is quite possible that in the unsaturated zone of the aquifer the capillary pressure never can rise up to the level of the entry pressure of the riverbed. The infiltration capacity of the unsaturated zone may reach its own limit.

As the water table drops and in order to sustain the seepage rate coming out of the riverbed, the interface capillary pressure must increase and overcome the capillary resistance. As the depth of the unsaturated zone increases that capillary resistance diminishes and become negligible. At the interface the flow is sustained by gravity at a rate equal to the relative conductivity, namely $K_V k_{rw}^{aq}(h_{cI})$. K_V is the aquifer vertical conductivity and $k_{rw}^{aq}(h_{cI})$ is the relative permeability in the aquifer. This must be able to sustain the seepage rate coming out of the riverbed, namely: $K[\frac{H+h_{cI}+e)}{e}$.

The limiting value of h_{cI} is thus the one for which there is equality for these two values:

$$K_V k_{rw}^{aq}(h_{cI}) = K[\frac{H+h_{cI}+e)}{e} \qquad (20)$$

In other words the stage where seepage reaches a maximum limit can be attained in the unsaturated zone of the aquifer without desaturation of the riverbed or may happen only if the riverbed itself desaturates. Which case will occur will depend heavily upon the respective entry pressures of the aquifer material and of the riverbed and their respective saturated hydraulic conductivities. Naturally the discussion here was for the case that the interface capillary pressure varies while the river stage itself remained constant. Since the river stage will tend to fluctuate one may never see a constant seepage rate.

5. CONCLUSIONS

We have been able to describe the phenomena of desaturation within the riverbed and the aquifer and their mutual interaction in a realistic and relatively simple manner. The algorithms could be inserted fairly easily into groundwater models and integrated hydrologic models for large-scale regional simulations, particularly if those models rely upon the numerical finite-difference formulation. The insertion of these procedures in models that use the finite-element formulation is more complicated. Currently the author is looking into the possibility of inserting that methodology in IWFM (Integrated Water Flow Model) of the California Department of Water Resources, which uses the finite-element formulation.

It is not only a problem of insertion but also of validation. The developed methods as described here and in earlier publications [1,2,4] are a definite improvement over earlier methods [3], from a physical and theoretical point of view. Since they rely upon approximations the actual practical merit of the new methods remains to be assessed and rigorously tested.

In addition here remains to investigate what happens when the river goes completely dry. In that case the riverbed drains simultaneously from the bottom and now also from the top.

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author is Emeritus Professor of Civil Engineering from Colorado State University.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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