

*Full Length Research Paper*

# Fixed lifetime inventory system with double order under useful lifetime based model

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**The problem of outdated in the fixed lifetime inventory system has been the focus of researchers in the last decade. Results show that while outdated is being minimized, shortages have become a problem. To eliminate or reduce shortages in the fixed lifetime inventory system, we propose a model where, there are two orders, one period apart in arriving into the inventory system. The second order is designed to satisfy any demand that cannot be satisfied by the first order.**

**Key words:** Outdating, shortages, fixed lifetime, order, inventory system.

## INTRODUCTION

One of the biggest problems of the fixed lifetime inventory system is outdated of products. Products are said to have expired (or outdated) if they have not been used to meet demand at the end of their useful lifetime in inventory. Many companies in Nigeria and Africa suffer huge financial losses annually due to outdated of fixed lifetime products. Besides the financial losses, there is the danger of citizens consuming outdated products which can lead to outbreak of diseases. The research focus [Chiu (1995), Liu and Lian (1999), Olsson and Tydesjo (2010), Nahmias (1982), Nahmias (2011), Silver et al. (2012), Mahmoodi et al (2015), Sheng-Chih et al. (2016), Izevbizua and Omosigho (2017), Izevbizua and Apanapudor (2019), Izevbizua and Emuefe (2019)] has been on how to reduce the quantity of items outdated in the fixed lifetime inventory system. While the outdate quantity was been minimized, shortages was observed to be on the increase. Many inventory managers have struggled with the problem of stocking because of outdated. Some have over stock and others under stock. Over stocking is having too many items on hand, leading

to an increase in outdated products, while under stocking is having too little on hand leading to shortages. The desired inventory policy is one that order in such a way that there will always be items on hand to meet demand, while reducing the outdate quantity. This is the main focus of this work namely; to develop inventory policies that will make goods available at all times and also reduce the amount of items outdated from the fixed lifetime inventory system. To do this, a double order fixed lifetime inventory model was introduced. The model consist of two orders which may or may not be equal in size and arrives into inventory one period apart (that is, the first order arrive in period  $i$  and the second order arrive in period  $i+1$ ). The second order is the backup order designed to take care of excess demand that cannot be satisfied by the first order. Items from the first order will outdate before items from the second order. The model allows us to trace items from the point of entry to the point of leaving the system either by demand or

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outdating.

**METHODOLOGY**

**Model assumptions**

- 1) Order is placed for new products when the useful lifetime remaining on the items on hand is one period.
- 2) There are two orders  $y_1$  and  $y_2$ . The orders arrive into inventory one period apart, that is if  $y_1$  arrives in period  $i$ ,  $y_2$  arrives in period  $i + 1$ .
- 3) Each ordered pair  $y_1$  and  $y_2$  forms a set of order.
- 4) The issuing policy is FIFO.
- 5) The age of items arriving into inventory is zero. Items not used to meet demand at the end of their useful periods outdate and are discarded.
- 6) Excess demand that cannot be satisfy from the back up order are lost.

**Notations**

- $y_1$  = first order of a set
- $y_2$  = second order of a set
- $y_{1,2}$  = order1 of set 2
- $y_{2,2}$  = order2 of set2
- $f(t)$  = demand density
- $x_1$  = reorder point wrt  $y_1$
- $x_2$  = reorder point wrt  $y_2$
- $m$  = lifetime of product
- $k$  = ordering cost
- $\theta$  = outdate cost
- $v$  = shortage cost
- $h$  = holding cost
- $d_i, i = 1,2,...,m$  = demand in period  $i$  for order  $y_i, i = 1,2$
- $t_i, i = 1,2,3,...,m$  = total demand in period  $i$
- $T_i$  = specific period in inventory

**Model description**

In Table 1,  $T_i$  represent specific periods in inventory. Each pair of orders  $y_1$  and  $y_2$  forms a set of order. The first order of set 1,  $y_1$  arrives at the start of period 1, age 0. It is depleted by the demand in period 1 and reduces to  $y_1 - d_1$ . The second order  $y_2$  arrives at the start of period 2. The two orders are further depleted by the demand in period 2. This movement from a lower period to a higher period continues for both orders until they get to period  $m$  which is the last useful period for the first order and period  $m + 1$  which is the last for the second order. At the end of period  $m$ , items from the first order expires and are discarded from inventory. Similarly, items from the second order expires at the of period  $m + 1$ .

Observe that when the useful lifetime remaining on items from the first order is one, a new order  $y_{1,2}$  (first order of set 2) arrives and

when the useful lifetime remaining on the second order is one, another order  $y_{2,2}$  (which is the second order of set 2) arrives. The new arrivals will undergo the same periodic movement until they expires, if not used to meet demand. In a fixed lifetime inventory system, it is necessary to track items of a particular order from the time they arrive in inventory to their last useful period in inventory. Table 2 shows the quantity of items on hand and their age distribution from Table 1.

Again, observe that at the end of period  $m + 1$ , no item(s) from the first set is left in the system. Either they have been used to meet demand or have expired. Next, we look at the amount of items on hand and their age categories. This enables us keep track of items from a particular order.

$$\begin{aligned}
 T_1 &= y_1^1, \text{ age } \{0\} \\
 T_2 &= (y_1^1 - d_{p_1}) + y_2^1, \text{ age } \{1,0\} \\
 T_3 &= (y_1^1 - d_{p_1} - d_{p_2}) + (y_2^1 - d_{p_1}), \text{ age } \{2,1\} \\
 &\vdots \\
 &\vdots \\
 T_{m-1} &= (x_1^1 - \sum_{i=1}^{m-1} d_{p_i}) + (y_2^1 - \sum_{i=1}^{m-2} d_{p_i}) + y_1^2, \text{ age } \{m-1, m-2, 0\} \\
 T_m &= (x_1^1 - \sum_{i=1}^m d_{p_i}) + (x_2^1 - \sum_{i=1}^{m-1} d_{p_i}) + (y_1^2 - d_{p_1}) + y_2^2, \text{ age } \{m, m-1, 1, 0\} \\
 T_{m+1} &= (x_2^1 - \sum_{i=1}^m d_{p_i}) + (y_1^2 - d_{p_1} - d_{p_2}) + (y_2^2 - d_{p_1}), \text{ age } \{m, 2, 1, \} \\
 T_{m+2} &= (x_1^2 - \sum_{i=1}^{m-1} d_{p_i}) + (y_2^2 - d_{p_1} - d_{p_2}) + y_1^3, \text{ age } \{m-1, 2, 0\}
 \end{aligned}$$

**RESULTS AND DISCUSSION**

**Derivation of total cost function for the model**

The total cost function for the model is the sum of the holding cost, shortage cost, outdate cost and ordering cost. That is;

$$\text{Total cost} = \text{ordering cost} + \text{shortage cost} + \text{outdate cost} + \text{holding cost}$$

**Ordering cost:** The ordering cost for the model is given as  $k(y_1 + y_2)$ , where  $y_1 + y_2$  is the total amount of items ordered in a set and  $k$  is the fixed ordering cost per unit ordered.

$$\text{Ordering cost} = k (y_1 + y_2) \tag{1}$$

**Shortage cost:** One of the advantage of the double order model is that shortages are highly minimized. This is because, the second order of each set act as a backup for the first order, by satisfying any demand that cannot be satisfied from the first order. However, if the demand in a period cannot be completely satisfied by items from both orders, the excess demand is lost. Such excess demand is referred to as shortages. Since total demand in a period is  $t$ , shortage occur if  $t > y_1 + y_2$ . So that our shortage quantity is given as

Table 1. Model.

Set	$T_1$	$T_2$	$T_3$	...	$T_{m-1}$	$T_m$	$T_{m+1}$			
1	$y_1$ age 0 ( $y_1 - t_1$ )	$y_1 - d_1 = a$ age 1	$y_1 - \sum_{i=1}^2 d_i = a$ age 2	...	$y_1 - \sum_{i=1}^{m-1} d_i = a$ age $m-1$	$y_1 - \sum_{i=1}^m d_i = a$ age $m$	$y_2 - \sum_{i=1}^m d_i = a$ age $m$			
		$y_2 = b$ age 0 ( $a + b - t_2$ )	$y_2 - d_1 = b$ age 1 ( $a + b - t_3$ )		$y_2 - \sum_{i=1}^{m-2} d_i = b$ age $m-2$	$y_2 - \sum_{i=1}^{m-1} d_i = b$ age $m-1$				
2					$T_1$ $y_{1,2} = c$ age 0 ( $a + b + c - t_{m-1}$ )	$T_2$ $y_{1,2} - d_1 = c$ age 1 $y_{2,2} = e$ age 0 ( $a + b + c + e - t_m$ )	$T_3$ $y_{1,2} - \sum_{i=1}^2 d_i = b$ age 2 $y_{2,2} - d_1 = c$ age 1 ( $a + b + c - t_{m+1}$ )	...	$T_{m-1}$	$T_m$

$$\text{Shortage quantity} = \int_{y_1+y_2}^{\infty} (t - (y_1 + y_2)) f(t) dt$$

And our shortage cost as

$$\text{Shortage cost} = v \int_{y_1+y_2}^{\infty} (t - (y_1 + y_2)) f(t) dt \quad (2)$$

**Outdate cost:** Outdate occurs when demand is low and items are not used to meet demand at the end of their useful periods in inventory. From Table 1, outdating will occur at the end of periods  $m$  (for items in  $y_1$ ) and period  $m+1$  (for items in  $y_2$ ). If the demand at the end of period  $m$  is less than  $x_1$  (items from the first order with only one useful period left on them at the start of period  $m$ ), then the outdate quantity with respect to  $y_1$  is given as

$$\text{Outdates from } y_1 = \int_0^{x_1} (x_1 - t) f(t) dt \quad (3)$$

Similarly, if the demand in period  $m+1$  is less than  $x_2$  (items from the second order with only one useful period

left on them at the start of period  $m+1$ ) then the outdate quantity with respect to the second order is given as;

$$\text{Outdates from } y_2 = \int_0^{x_2} (x_2 - t) f(t) dt \quad (4)$$

Hence, the total outdate quantity from the set is given as;

$$\text{outdates from the set} = \int_0^{x_1} (x_1 - t) f(t) dt + \int_0^{x_2} (x_2 - t) f(t) dt \quad (5)$$

And the outdate cost for the model is

$$\text{outdates cost for the set} = \theta \left\{ \int_0^{x_1} (x_1 - t) f(t) dt + \int_0^{x_2} (x_2 - t) f(t) dt \right\} \quad (6)$$

**Holding cost:** There is a fixed cost  $h > 0$  for each item(s) held in inventory, so that our holding cost is;

$$\text{Holding cost} = h \int_0^{y_1+y_2} (y_1 + y_2 - t) f(t) dt \quad (7)$$

Therefore, our total cost function is the sum of all the cost components.

**Table 2.** Quantity of items on hand and their ages at the start of the periods.

Period	Quantity of items on hand	Age(s)
1	$y_1$	0
2	$y_1 - d_1$	1
	$y_2$	0
3	$y_1 - d_1 - d_2$	2
	$y_2 - d_1$	1
	⋮	
	⋮	
	⋮	
$m-1$	$y_1 - \sum_{i=1}^{m-1} d_i$	$m-1$
	$y_2 - \sum_{i=1}^{m-2} d_i$	$m-2$
	$y_{1,2}$	0
$m$	$y_1 - \sum_{i=1}^m d_i$	$m$
	$y_2 - \sum_{i=1}^{m-1} d_i$	$m-1$
	$y_{1,2} - d_1$	1
	$y_{2,2}$	0
$m+1$	$y_2 - \sum_{i=1}^m d_i$	$m$
	$y_{1,2} - \sum_{i=1}^2 d_i$	2
	$y_{2,2} - d_1$	1
$m+2$	$y_{1,2} - \sum_{i=1}^3 d_i$	3
	$y_{2,2} - \sum_{i=1}^2 d_i$	2

$$C(y_1, y_2, x_1, x_2) = k(y_1 + y_2) + h \int_0^{y_1+y_2} (y_1 + y_2 - t)f(t)dt + \theta \left\{ \int_0^{x_1} (x_1 - t)f(t)dt + \int_0^{x_2} (x_2 - t)f(t)dt \right\} + v \int_{y_1+y_2}^{\infty} (t - (y_1 + y_2))f(t)dt \tag{8}$$

**Numerical example**

Table 3 gives the quantity of items ordered in a departmental shop in Benin City, Nigeria, the demand for each order, the shortage and outdates associated with each order.

At the end of 30 days of applying the ordering policy of the model, the number of shortages was zero while the

A computer programme in MATHEMATICA 8 was used to solve the total cost function in Equation 8.

**Table 3.** Orders, demand, shortage and outdates for a product with 4useful-lifetime and zero lead time.

Day	$y_1$	$y_2$	Demand	Shortage	Outdates
1	$y_1^1 = 70$	0	20	-	-
2	50	$y_2^1 = 25$	40	-	-
3	10 $y_1^2 = 20$	25	32	-	-
4	20	3 $y_2^2 = 38$	29	-	-
5	0 $y_1^3 = 50$	32	30	-	-
6	50	2 $y_2^3 = 25$	15	-	-
7	37 $y_1^4 = 30$	25	17	-	-
8	20 30	25 $y_2^4 = 20$	15	-	5
9	30 $y_1^5 = 10$	25 20	40	-	-
10	15 10	20 $y_2^5 = 10$	31	-	-
11	10 $y_1^6 = 40$	4 10	38	-	-
12	26	$y_2^6 = 45$	30	-	-
13	0 $y_1^7 = 45$	41	50	-	-
14	36	0 $y_2^7 = 45$	28	-	-
15	8 $y_1^8 = 39$	45	30	-	-
16	39	23 $y_2^8 = 35$	25	-	-
17	37 $y_1^9 = 37$	35	15	-	-
18	22 37	35 $y_2^9 = 15$	18	-	4
19	37 $y_1^{10} = 20$	35 15	48	-	-
20	24 20	15 $y_2^{10} = 20$	44	-	-

**Table 3.** Contd.

21	15 $y_1^{11} = 35$	20	25	-	-
22	35	10 $y_2^{11} = 35$	40	-	-
23	5 $y_1^{12} = 30$	35	35	-	-
24	30	5 $y_2^{12} = 25$	30	-	-
25	5 $y_1^{13} = 40$	25	40	-	-
26	30	$y_2^{13} = 45$	35	-	-
27	0 $y_1^{14} = 45$	40	38	-	-
28	45	2 $y_2^{14} = 30$	37	-	-
29	10 $y_1^{15} = 30$	30	32	-	-
30	30	8 $y_2^{15} = 35$	40	-	-

**Table 4.** Double orders and associated cost for  $m=4$  and  $l = 0$ .

Set	$y_1$	$y_2$	$x_1$	$x_2$	Outdates	Demand	Costs
1	70	25	10	3	-	95	22970.87
2	20	38	0	2	-	58	6356.89
3	50	25	37	25	5	70	57114.95
4	30	20	30	20	-	50	33510
5	10	0	10	0	-	20	1633.41
6	40	45	0	0	-	85	12191.67
7	45	45	8	23	-	90	32979.26
8	39	35	37	35	4	70	70638.19
9	37	15	37	15	-	52	36215.02
10	20	20	15	10	-	40	11596.96
11	35	35	5	5	-	70	12340.79
12	30	25	5	0	-	55	6695.45
13	40	45	0	2	-	85	13061.83
14	45	30	10	8	-	75	18025.77
15	30	35					

outdates was 9. The problem of shortages have been addressed by the model as the second order mops up excess demand.

From Table 3, Table 4 is obtained showing the cost associated with each set of orders. Constant parameters are  $k = 150$ ,  $v = 10$ ,  $h = 20$ ,  $\theta = 0.005$ .

## Conclusion

The double order model reduces the shortages associated with the fixed lifetime inventory system. The second order act as backup order, meeting demands that cannot be satisfied by the first order. This will restore customers' confidence in the inventory manager. The double order alongside the ordering policy based on the remaining useful lifetime have the advantage of making goods available and minimizing outdated as shown in the numerical example.

## CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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