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Hotelling's Rule Verification of a Mining Project

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

On the basis of a real (confidential) mining project data the Hotelling's rule fitting is verified and a new production schedule is proposed, which better fulfill the Hotelling rate rule than the previously program published by the investor. The intertemporal influences in project duration period are analyzed.

Keywords: Calculus of variations; mining; economic profit.

1. INTRODUCTION

The economic analysis of the industry based on the exploitation of mineral resources, differs fundamentally from the analyzes in agriculture, manufacturing industry and services [1,2]. The main reason of this differences is that the mineral resources are exhaustible resources. In other words, in the extractive industry an initial stock of reserves will run out over time by exploitation.

If we start from the prerequisite that the owner of a mineral resource, the same as any other owner, is looking for the maximum profit, then this one too it must also take into account certain factors specific to the extractive industry [3-5]. In other economic sectors, the profit is maximized when the marginal cost is equal to the marginal income.

When taking a decision to start the extracting of a reserve, the owner of the resource must be satisfied not only with the fact that current profit increase, yet also with the fact that this increase is more important than the future reduction of profit, pr it may delay the extraction.

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The economy of the extractive industry claim that the incomes which could appear from the current extraction should be big enough to cover the marginal cost of extraction as well as the marginal cost of the user [6-8].

This leads us to the known law as the "fundamental principle" of the extractive industry or Hotelling`s rule.

2. WORKING METHODS

This means that in order for the extraction to be justified the net marketing price of the resource (extraction net cost) must increase in a linear way with the market profit rate.

Formally, the fundamental principle can be obtained from a series of simplifying hypotheses:

- The owner of the resource has a fixed stock of non-renewable resources and wants to deplete the reserves at a rate that gives him maximum profit, meaning that the owner wants to maximize the present value of his income which results from the extraction over time of his reserves;
- The quality of the resource is uniform throughout the extraction;
- The extraction rate can be easily modified without losses ;
- The extraction cost is constant over time.

In these conditions the function of the profit which must be maximized is:

$$max \int_0^T \pi e^{-\gamma t} dt = \int_0^T [P(t)Q(t) - C(t)Q(t)] e^{-\gamma t} dt \quad (1)$$

under the conditions of a constraint stock:

$$\int_0^T Q(t) dt = \bar{Q} \quad (2)$$

In which:

π = the function of the updated profit;
P(t) = the price of the resource at time t;
C = cost of extraction, constant;
Q(t) = the quantity extracted at time t;
t = time, in years;
T = the number of years of exploitation of the reserve, for example, the depletion of the reserve;
γ = the update rate;
$\bar{Q}$ – the total stock (reserve).

The expression (1) is a functional and represents the net profit updated during the operating period of the reserve.

What needs to be determined is the programming of the annual production in order to achieve a maximum profit in conditions of uncertainty, because in the long period of the project, the economic situations – the price difference – costs can fluctuate.

For maximization, the Lagrange multipliers method is used and the functional

(1) becomes:

$$\int_0^T [P(t)Q(t) - C(t)Q(t)] e^{-\gamma t} + \lambda(\bar{Q} - \int_0^T Q(t)) dt) \quad (3)$$

Equaling with zero the first variation:

$$\frac{\delta L}{\delta Q} = [P(t) - C(t)Q] e^{-\gamma t} - \lambda = 0 \quad (4)$$

the following relation is obtained:

$$P(t) - C(t) = \lambda (e)^{\gamma t} \quad (5)$$

The left side of the equality represents (the net price of the reserve – the net cost of extraction) and the right side is the rent of the resource.

The above equation shows that the price minus the cost of extraction grows linearly with the market rate of profit.

Writing the equation under the form:

$$[P(t) - C(t)] e^{-\gamma t} = \lambda \quad (6)$$

results that the ratio between the net price of the resource and the profit rate at any time is a constant.

This formulation is also called the Hotelling rule, and the value λ is called the Hotelling rent.

Analyzing Fig. 1, that represents the foresight of the annual exploitation rate of a mining project, we can observe that in the initial version of the project, proposed by the investors, the exploitation rate of the mining mass varies approximately after a „bell curve”, according to Hubert`s theory.

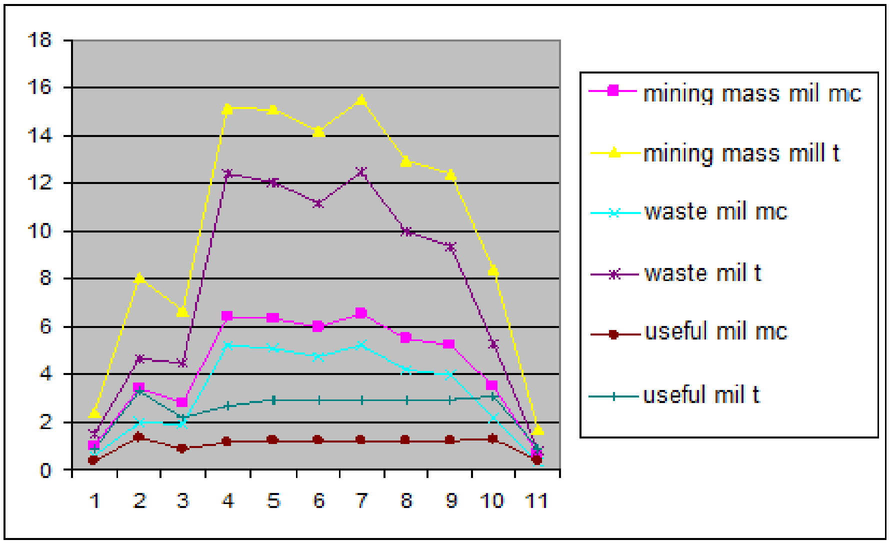

Fig. 1. The career production capacity, staggered by years

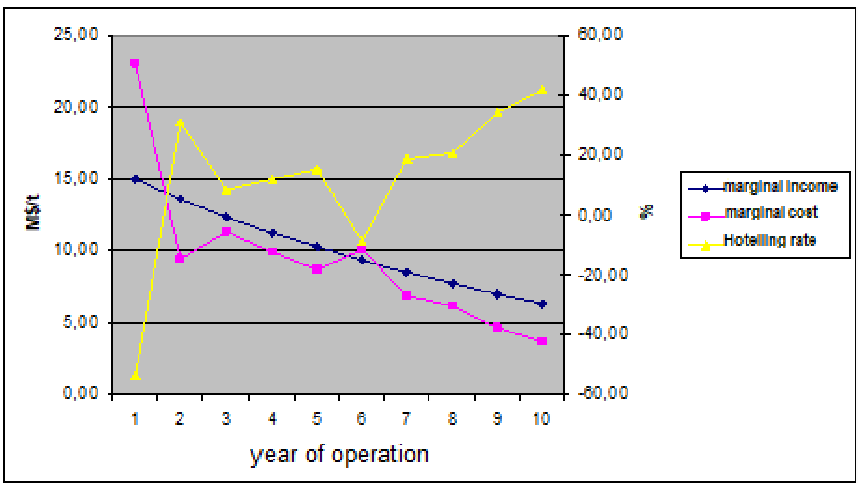

Fig. 2. The graphical representation of Hotelling condition for the variant version proposed by the investor

However, the amount of metal extracted is constant, around 3 tons/year, being equal to the refining capacity of the preparation plant.

From the start, it can be estimated even intuitively that this variation over time of the exploitation rate is not sustainable, leading to depletion of the reserve at a constant rate.

The verification of the sustainability of the project according to Hotelling`s criteria, whose results are presented in Fig. 2, demonstrates that in the version proposed by the investor, the Hotelling`s natural capital devaluation rate, corresponding to λ from ecuation (5) (yellow curve), it is not constant.

The capital depreciation rate λ is expressed by the difference between the marginal income and the marginal cost, reported to marginal income in each period (year) of production.

To optimize the ore extraction, I realized and used a calculus application in EXCEL to readjust the annual production values and staggering the investments by years, so that the capital depreciation rate should equalize to 10%.

Optimized version

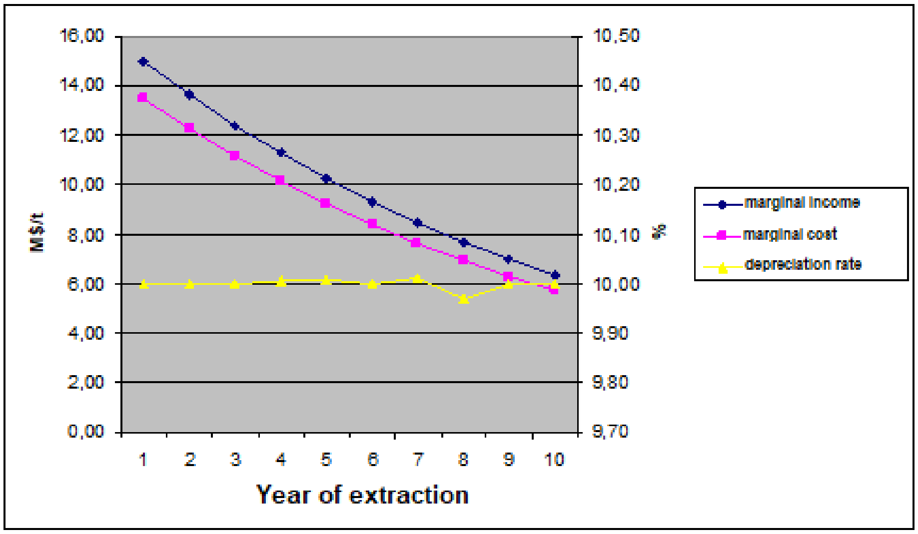

Fig. 3. Graphical representation of the Hotelling condition for the optimized version

3. RESULTS AND DISCUSSION

The proposed solution after verification and optimization is presented in Fig. 3.

This verification of the sustainability of a mining project by the Hotelling`s method proves that the project can be adjusted, with the same investments, but in different stages, to ensure at all times an equal option for abandoning the project in favor of another more profitable investment due to the depreciation of the residual stock value.

If worldwide the applications of some advanced chapters in mathematics in various fields of engineering have recorded an explosive dynamics in the recent years in the mining industry, known as reluctant to progress and at the same time hungry for technological innovation, yet burdened by intrinsic endemic difficulties and inherent, resulting spectacular results that appear sporadically. The scientific support from the academic environment being strongly felt.

The aproached topic in this paper, which starts from the needs of industry and the possibilities, some neglected, of mathematics, whether if we are talking about analytical or numerical instruments in solving this cognitive crisis, is timely and current.

In order to illustrate the value of analysis based on variational methods, in solving technical, economical and multidisciplinary problems, from the mining and geotechnics area, where the problem of optimizing mathematically descriptive processes by functional, I applied the calculus of variations and related tools.

Table 1. Calculus of the Hotelling relation values for the optimized version

Year	1	2	3	4	5	6	7	8	9	10
Mining Mass, Mt	8,9	7,0	15,5	15,1	14,0	15,7	12,5	12,3	7,9	1,8
Waste , Mt	4,7	4,5	12,4	12,1	11,2	12,5	10,0	9,4	5,3	0,8
Useful, Mt	4,2	2,5	3,1	3,0	2,8	3,2	2,5	2,9	2,6	1,0
Metal, t	8,5	4,9	6,2	6,0	5,5	6,4	5,0	5,8	5,3	2,1
Outgoing, M$	45	20	3	2	2	4	2	10	20	12
Marginal income, $/t	14,99	13,63	12,39	11,26	10,24	9,31	8,46	7,69	6,99	6,36
Marginal cost, $/t	13,49	12,27	11,15	10,14	9,21	8,38	7,62	6,93	6,29	5,72
Depreciation rate	10,00	10,00	10,00	10,00	10,01	10,00	10,01	9,97	10,00	10,00

4. CONCLUSION

This application refers to the analysis of evaluating the sustainability of a mining project, using the Hotelling`s rule method, which I treated as a variational problem of maximizing the net profit. Applying the method to a real project, I proved that the project can be adjusted with the same investments, otherwise staggered to ensure at any time an equal option for abandoning the project, in favor of another more profitable investment due to the depreciation of the residual value of the deposit or continuation of exploitation under favorable conditions.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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